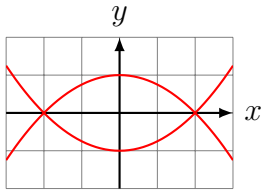


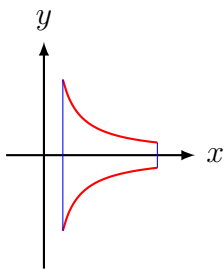
Aufgabe 7.1



Nullstellen: $\frac{1}{4}(x^2 - 4) = 0 \Rightarrow x = \pm 2$

$$\begin{aligned} V &= 2\pi \int_0^2 \left[\frac{1}{4}(x^2 - 4) \right]^2 dx \\ &= \frac{1}{8}\pi \int_0^2 (x^4 - 4x^2 + 16) dx \\ &= \frac{1}{8}\pi \left[\frac{1}{5}x^5 - \frac{4}{3}x^3 + 16x \right]_0^2 = \frac{32}{15}\pi \end{aligned}$$

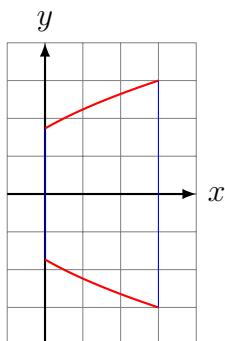
Aufgabe 7.2



$$\begin{aligned} V &= \pi \int_a^b (x^{-1})^2 dx = \pi \int_a^b (x^{-2}) dx = \pi [-x^{-1}]_a^b \\ &= \pi \left(-\frac{1}{b} + \frac{1}{a} \right) = \pi \left(\frac{1}{a} - \frac{1}{b} \right) \end{aligned}$$

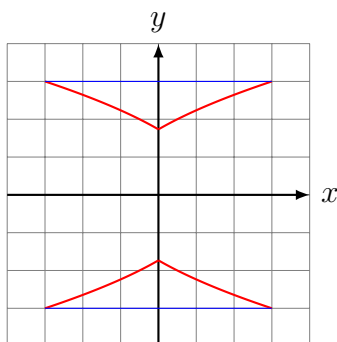
Aufgabe 7.3

$$(a) \quad y^2 = 2x + 3 \quad \Leftrightarrow \quad y = \pm\sqrt{2x + 3}$$



$$\begin{aligned} V_x &= \pi \int_1^3 (2x + 3) \, dx = \pi \int_1^3 (x^2 + 2x + 1) \, dx \\ &= \pi [x^2 + 2x + 1]_1^3 = 18\pi \end{aligned}$$

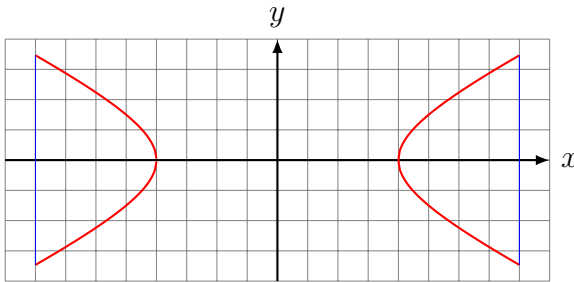
$$(b) \quad y^2 = 2x + 3 \quad \Leftrightarrow \quad x = \frac{1}{2}(y^2 - 3)$$



$$\begin{aligned} V_y &= 2\pi \int_{\sqrt{3}}^3 \frac{1}{4}(y^2 - 3)^2 \, dy = \frac{\pi}{2} \int_{\sqrt{3}}^3 (y^4 - 6y^2 + 9) \, dy \\ &= \frac{\pi}{2} \left[\frac{1}{5}y^5 - 2y^3 + 9y \right]_{\sqrt{3}}^3 \\ &= \frac{\pi}{2} \left[\left(\frac{243}{5} - 54 + 27 \right) - \left(\frac{9\sqrt{3}}{5} - 6\sqrt{3} + 9\sqrt{3} \right) \right] \\ &= \frac{\pi}{2} \left[\frac{108}{5} - \frac{24\sqrt{3}}{5} \right] \\ &= \frac{\pi}{5} (54 - 12\sqrt{3}) = \frac{6\pi}{5} (9 - 2\sqrt{3}) \end{aligned}$$

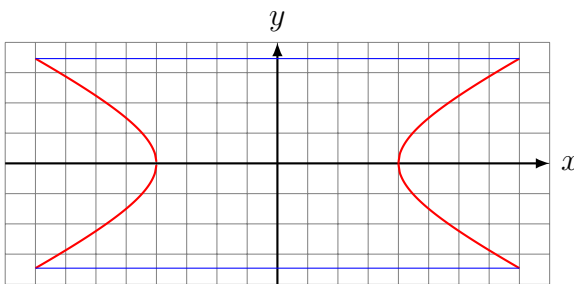
Aufgabe 7.4

(a) $x^2 - 4y^2 = 16 \Leftrightarrow y^2 = \frac{1}{4}(x^2 - 16) \quad (4 \leq x \leq 8)$



$$\begin{aligned} V_x &= 2\pi \int_4^8 \frac{1}{4}(x^2 - 16) dx = \frac{\pi}{2} \left[\frac{1}{3}x^3 - 16x \right]_4^8 \\ &= \frac{\pi}{2} \left[\left(\frac{512}{3} - 128 \right) - \left(\frac{64}{3} - 64 \right) \right] = \frac{\pi}{2} \cdot \frac{256}{3} = \frac{128}{3}\pi \end{aligned}$$

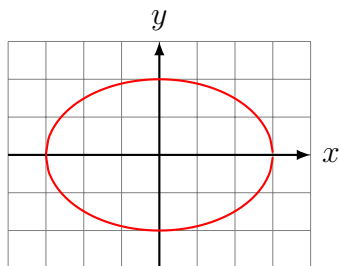
(b) $x^2 - 4y^2 = 16 \Leftrightarrow x^2 = \sqrt{16 + 4y^2}$



$$\begin{aligned} V_x &= 2\pi \int_0^{2\sqrt{3}} (16 + 4y^2) dy = 2\pi \left[16y + \frac{4}{3}y^3 \right]_0^{2\sqrt{3}} \\ &= 2\pi \left[32\sqrt{3} + 32\sqrt{3} \right] = 128\sqrt{3}\pi \end{aligned}$$

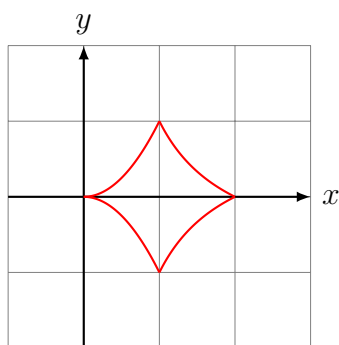
Aufgabe 7.5

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \Rightarrow \quad y^2 = b^2 - \frac{b^2}{a^2}x^2$$



$$\begin{aligned} V_x &= 2\pi \int_0^a \left(b^2 - \frac{b^2}{a^2}x^2 \right) dx = 2b^2\pi \left[x - \frac{1}{3a^2}x^3 \right]_0^a \\ &= 2b^2\pi \left[a - \frac{1}{3}a \right] = \frac{4}{3}ab^2\pi \end{aligned}$$

Aufgabe 7.6



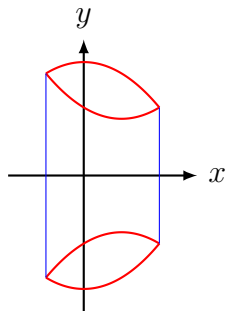
Schnittstelle: $x^2 = 2/x - 1$
 $x^3 = 2 - x$
 $x^3 + x - 2 = 0$
 $x = 1$ (einzige reelle Nullstelle)

$$V_1 = \pi \int_0^1 (x^2)^2 dx = \pi \int_0^1 x^4 dx = \pi \left[\frac{1}{5}x^5 \right]_0^1 = \frac{1}{5}\pi$$

$$\begin{aligned} V_2 &= \pi \int_1^2 (2x^{-1} - 1)^2 dx = \pi \int_1^2 (4x^{-2} - 4x^{-1} + 1) dx \\ &= \pi \left[-4x^{-1} - 4\ln(x) + x \right]_1^2 \\ &= \pi \left[(-2 - 4\ln(2) + 2) - (-4 - 0 + 1) \right] \\ &= \pi [3 - 4\ln(2)] \end{aligned}$$

$$V = V_1 + V_2 = \pi [3.2 - 4\ln(2)]$$

Aufgabe 7.7



Schnittstellen: $f(x) = g(x)$

$$x^2 - 2x + 6 = -x^2 + 10$$

$$2x^2 - 2x - 4 = 0$$

$$x^2 - x - 2 = 0$$

$$x_1 = -1$$

$$x_2 = 2$$

Äusseres Volumen:

$$V_1 = \pi \int_{-1}^2 (10 - x^2)^2 dx$$

$$= \pi \int_{-1}^2 (100 - 20x^2 + x^4) dx$$

$$= \pi \left[100x - \frac{20}{3}x^3 + \frac{1}{5}x^5 \right]_{-1}^2 = \frac{1233}{5}\pi$$

Inneres Volumen:

$$V_2 = \pi \int_{-1}^2 (x^2 - 2x + 6)^2 dx$$

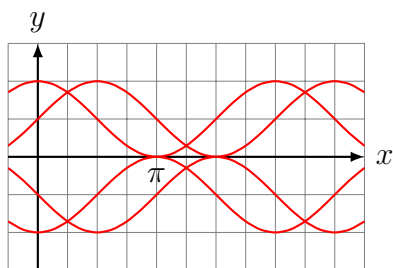
$$= \pi \int_{-1}^2 (x^4 + 4x^2 + 36 - 4x^3 - 24x + 12x^2) dx$$

$$= \pi \int_{-1}^2 (x^4 - 4x^3 + 16x^2 - 24x + 36) dx$$

$$= \pi \left[\frac{1}{5}x^5 - x^4 + \frac{16}{3}x^3 - 12x^2 + 36x \right]_{-1}^2 = \frac{558}{5}\pi$$

$$\text{Differenz: } V = V_2 - V_1 = \frac{1233}{5}\pi - \frac{558}{5}\pi = \frac{675}{5}\pi = 135\pi$$

Aufgabe 7.8



Schnittstellen: $1 + \sin(x) = 1 + \cos(x)$

$$\sin(x) = \cos(x)$$

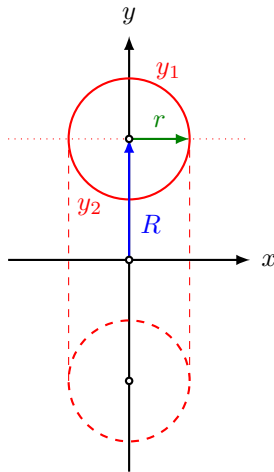
$$\frac{\sin(x)}{\cos(x)} = 1$$

$$\tan(x) = 1$$

$$x_k = \frac{\pi}{4} + k\pi$$

$$\begin{aligned} V_2 &= \pi \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (1 + \sin x)^2 dx \\ &= \pi \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (1 + 2\sin x + \sin^2 x) dx \\ &= \pi \left[x - 2\cos x + \frac{1}{2}x - \frac{1}{4}\sin 2x \right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \\ &= \pi \left[\frac{3}{2}x - 2\cos x - \frac{1}{4}\sin 2x \right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \\ &= \pi \left[\frac{3}{2} + 2\sqrt{2} \right] \\ V_1 &= \pi \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (1 + \cos x)^2 dx \\ &= \pi \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (1 + 2\cos x + \cos^2 x) dx \\ &= \pi \left[x - 2\cos x + \frac{1}{2}x - \frac{1}{4}\sin(2x) \right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \\ &= \pi \left[\frac{3}{2} - 2\sqrt{2} \right] \\ V &= V_2 - V_1 = 4\sqrt{2}\pi \end{aligned}$$

Aufgabe 7.9



$$y_1 = R + \sqrt{r^2 - x^2} \quad \text{oberer Halbkreis}$$

$$y_2 = R - \sqrt{r^2 - x^2} \quad \text{unterer Halbkreis}$$

$$V_1 = 2\pi \int_0^r (R + \sqrt{r^2 - x^2})^2 dx$$

$$V_2 = 2\pi \int_0^r (R - \sqrt{r^2 - x^2})^2 dx$$

$$V = V_1 - V_2$$

$$= 2\pi \int_0^r [(R^2 + 2R\sqrt{*} + \sqrt{*}^2) - (R^2 - 2R\sqrt{*} + \sqrt{*}^2)] dx$$

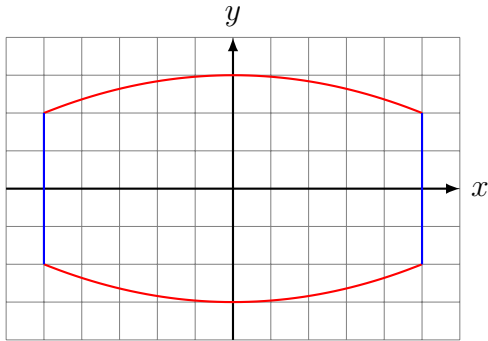
$$= 8\pi R \int_0^r \sqrt{r^2 - x^2} dx$$

$$= 8\pi R \left[\frac{x}{2} \sqrt{r^2 - x^2} + \frac{r^2}{2} \arcsin \frac{x}{r} \right]_0^r$$

$$= 8\pi R \left[\left(0 + \frac{r^2}{2} \arcsin 1 \right) - \left(0 + \frac{r^2}{2} \arcsin 0 \right) \right]$$

$$= 8\pi R \cdot \frac{r^2}{2} \cdot \frac{\pi}{2} = 2\pi^2 R r^2$$

Aufgabe 7.10



$$f(x) = ax^2 + b$$

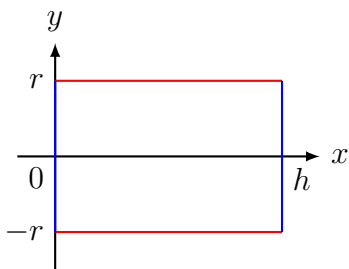
$$f(0) = 3 = b$$

$$f(5) = 2 = 25a + 3 \quad \Rightarrow \quad a = -\frac{1}{25}$$

$$f(x) = 3 - \frac{1}{25}x^2$$

$$\begin{aligned} V &= 2\pi \int_0^5 \left(3 - \frac{1}{25}x^2\right)^2 dx \\ &= 2\pi \int_0^5 \left(9 - \frac{6}{25}x^2 + \frac{1}{625}x^4\right) dx \\ &= 2\pi \left[9x - \frac{2}{25}x^3 + \frac{1}{3125}x^5\right]_0^5 \\ &= 2\pi[45 - 10 + 1] = 72\pi \text{ dm}^3 \approx 226.19 \text{ Liter} \end{aligned}$$

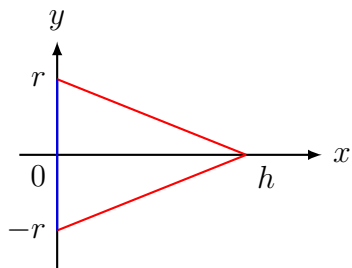
Aufgabe 7.11



$$f(x) = r \quad \Rightarrow \quad f'(x) = 0$$

$$M = 2\pi \int_0^h r\sqrt{1+0^2} dx = 2\pi \int_0^h r dx = 2\pi r[x]_0^h = 2\pi rh$$

Aufgabe 7.12



$$f(x) = -\frac{r}{h}x + r \quad \Rightarrow \quad f'(x) = -\frac{r}{h}$$

$$M = 2\pi \int_0^h \left(r - \frac{r}{h}x\right) \sqrt{1 + \left(-\frac{r}{h}\right)^2} dx$$

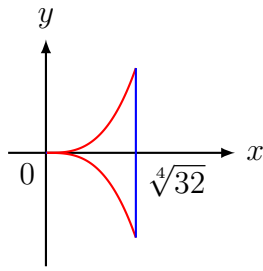
$$= 2\pi \sqrt{\frac{h^2 + r^2}{h^2}} \int_0^h \left(r - \frac{r}{h}x\right) dx$$

$$= 2\pi \sqrt{\frac{m^2}{h^2}} \left[rx - \frac{r}{2h}x^2\right]_0^h$$

$$= 2\pi \frac{m}{h} \left(rh - \frac{r}{2h}h^2\right)$$

$$= 2\pi \frac{m}{h} \cdot \frac{rh}{2} = \pi r m$$

Aufgabe 7.13



$$f(x) = \frac{1}{6}x^3 \quad \Rightarrow \quad f'(x) = \frac{1}{2}x^2$$

$$M = 2\pi \int_0^{\sqrt[4]{32}} \frac{1}{6}x^3 \sqrt{1 + \frac{1}{4}x^4} dx = \dots$$

$$\text{Substitution: } u = 1 + \frac{1}{4}x^4$$

$$\frac{du}{dx} = x^3 \quad \Rightarrow \quad dx = \frac{1}{x^3} du$$

$$\text{Grenzen: } u(0) = 1 + 0 = 1$$

$$u(\sqrt[4]{32}) = 1 + 8 = 9$$

$$\dots = 2\pi \int_1^9 \frac{1}{6}x^3 \sqrt{u} \frac{1}{x^3} du$$

$$= \frac{1}{3}\pi \int_1^9 \sqrt{u} du$$

$$= \frac{2}{9}\pi [u^{3/2}]_1^9 = \frac{2}{9}\pi(27 - 1) = \frac{52}{9}\pi$$