Aufgabe 10.1

$f^{(0)}(x) = \cos(x) \quad \Rightarrow \quad f^{(0)}(\pi) = \cos(\pi) = -1$

$f^{(1)}(x) = -\sin(x) \quad \Rightarrow \quad f^{(1)}(\pi) = -\sin(\pi) = 0$

$f^{(2)}(x) = -\cos(x) \quad \Rightarrow \quad f^{(2)}(\pi) = -\cos(\pi) = 1$

$f^{(3)}(x) = \sin(x) \quad \Rightarrow \quad f^{(3)}(\pi) = \sin(\pi) = 0$

$f^{(4)}(x) = \cos(x) \quad \Rightarrow \quad f^{(4)}(\pi) = \cos(\pi) = -1$

$$\cos(x) \approx -1 + \frac{1}{2!}(x-\pi)^2 - \frac{1}{4!}(x-\pi)^4$$

Aufgabe 10.2

(a) $f^0(x) = \dfrac{1}{x+1} \quad \Rightarrow \quad f^0(0) = 1$

$f^1(x) = -\dfrac{1}{(1+x)^2} \quad \Rightarrow \quad f^1(0) = -1$

$$\frac{1}{x+1} \approx 1 - x$$

(b) Formelsammlung (S. 64):

- exakter Wert: $a = \dfrac{1}{0.01+1} = \dfrac{1}{101/100} = \dfrac{100}{101}$
- Näherung: $a_0 = 1 - 0.01 = 0.99 = \dfrac{99}{100}$
- absoluter Fehler: $\varepsilon_a = a_0 - a = \dfrac{99}{100} - \dfrac{100}{101} = \dfrac{-1}{10100}$
- relativer Fehler: $\dfrac{\varepsilon_a}{a} = \dfrac{101}{100} \cdot \dfrac{-1}{101\,000} = -10^{-4}$

Aufgabe 10.3

(a) $\dfrac{\sin x - x\cos x}{x^2}$

$$= \frac{1}{x^2}\left[\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots\right) - x\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots\right)\right]$$

$$= \frac{1}{x^2}\left[-\frac{x^3}{3!} + \frac{x^3}{2!} + \frac{x^5}{5!} - \frac{x^5}{4!} - \dots\right]$$

$$= \left[-\frac{x}{3!} + \frac{x}{2!} + \frac{x^3}{5!} - \frac{x^3}{4!} - \dots\right] \to 0 \quad \text{wenn } x \to 0$$

Aufgabe 10.3

(b) $$\frac{\ln(1+x)}{x} = \frac{1}{x} \cdot \left[x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \right]$$

$$= 1 - \frac{x}{2} + \frac{x^2}{3} - \dots \to 1 \quad \text{für } x \to 0$$

Aufgabe 10.4

$$\arcsin(x) = \mathrm{e}^x$$

$$x \approx 1 - x$$

$$2x \approx 1$$

$$x \approx 0.5$$

Aufgabe 10.5

$$p(1+x) = p^{(0)}(1) + p^{(1)}(1)x + \frac{p^{(2)}(1)}{2!}x^2 + \frac{p^{(3)}(1)}{3!}x^3 + \frac{p^{(4)}(1)}{4!}x^4$$

$$p^{(0)}(x) = x^4 + 2x^3 - x + 4 \quad \Rightarrow \quad p^{(0)}(1) = 6$$

$$p^{(1)}(x) = 4x^3 + 6x^2 - 1 \quad \Rightarrow \quad p^{(1)}(1) = 9$$

$$p^{(2)}(x) = 12x^2 + 12x \quad \Rightarrow \quad p^{(2)}(1) = 24$$

$$p^{(3)}(x) = 24x + 12 \quad \Rightarrow \quad p^{(3)}(1) = 36$$

$$p^{(4)}(x) = 24 \quad \Rightarrow \quad p^{(4)}(1) = 24$$

$$p(x+1) = 6 + 9x + \frac{24}{2!}x^2 + \frac{36}{3!}x^3 + \frac{24}{4!}x^4$$

$$= x^4 + 6x^3 + 12x^2 + 9x + 6$$