
Trigonometrie
Aufgaben (L+)

Aufgabe 1.1

$$\left(14 + \frac{27}{60} + \frac{10}{3600}\right)^\circ = 14.4528^\circ$$

Aufgabe 1.2

$$0.1253 \cdot 60' = 7.518'$$

$$0.518 \cdot 60'' = 31.08''$$

$$47.1253^\circ = 47^\circ 7' 31.08''$$

Aufgabe 1.3

Erdumfang: $u = 2\pi \cdot r_{\text{Erde}} \approx 2\pi \cdot 6370 \text{ km} \approx 40\,000 \text{ km}$

360° entspricht etwa 40 000 km

1° entspricht etwa 111 km

$1'$ entspricht etwa 1.85 km

$1''$ entspricht etwa 0.031 km = 31m

Aufgabe 1.4

$$\left(137 + \frac{45}{60} + \frac{36}{3600}\right)^\circ = 137.76^\circ$$

$$\frac{137.76^\circ \cdot 400^g}{360^\circ} = 153.0667^g$$

Aufgabe 1.5

$$\beta = 212.5^g = \frac{212.5^g \cdot 360^\circ}{400^g} = 191.25^\circ = 191^\circ 15'$$

Aufgabe 1.6

$$\alpha = \frac{7\pi}{15} \cdot \frac{180^\circ}{\pi} = 84^\circ$$

Aufgabe 1.7

$$\beta = 25.2^\circ \cdot \frac{\pi}{180^\circ} = 0.44 \text{ rad}$$

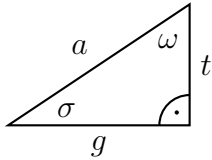
Aufgabe 1.8

$$\gamma = 302.5^g \cdot \frac{2\pi}{400^g} = 1.5125\pi \text{ rad} \approx 4.7517 \text{ rad}$$

Aufgabe 1.9

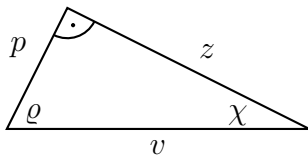
$$\delta = 4.13 \text{ rad} \cdot \frac{200^{\text{g}}}{\pi} \approx 262.92^{\text{g}}$$

Aufgabe 2.1



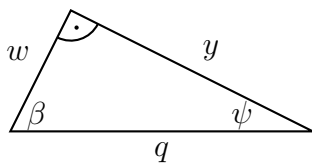
- (a) t ist die Ankathete von ω .
- (b) a ist die Hypotenuse.
- (c) t ist die Gegenkathete von σ

Aufgabe 2.2



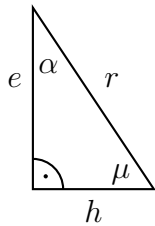
- (a) z ist Ankathete von χ und Gegenkathete von ϱ .
- (b) p ist Ankathete von ϱ und Gegenkathete von χ .
- (c) v ist die Hypotenuse.

Aufgabe 2.3



- (a) $\tan(\beta) = \frac{\text{GK}}{\text{AK}} = \frac{y}{w}$
- (b) $\cos(\psi) = \frac{\text{AK}}{\text{Hyp}} = \frac{y}{q}$
- (c) $\sin(\beta) = \frac{\text{GK}}{\text{Hyp}} = \frac{y}{q}$

Aufgabe 2.4



$$(a) \cos(\mu) = \frac{AK}{\text{Hyp}} = \frac{h}{r}$$

$$(b) \tan(\mu) = \frac{GK}{AK} = \frac{e}{h}$$

$$(c) \cos(\alpha) = \frac{GK}{\text{Hyp}} = \frac{e}{r}$$

Aufgabe 2.5

Berechne mit dem Taschenrechner auf 4 signifikante Stellen:

$$(a) \sin(2.5^\circ) = 0.04362 \text{ [mode: DEGREE]}$$

$$(b) \cos(1.1) = 0.4536 \text{ [mode: RADIAN]}$$

$$(c) \tan(40^g) = \tan(40/400 \cdot 360) = 0.7265 \text{ [mode: DEGREE]}$$

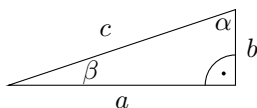
Aufgabe 2.6

$$(a) \arcsin(0.47) = 0.4893 \text{ rad [mode: RADIAN]}$$

$$(b) \arctan(2.94) = 71.21^\circ \text{ [mode: DEGREE]}$$

$$(c) \arccos\left(\frac{\sqrt{5}+1}{4}\right) = 36^\circ = \frac{36^\circ \cdot 400^g}{360^\circ} = 40^g$$

Aufgabe 3.1

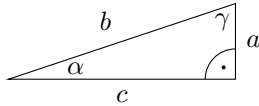


$$\alpha = 180^\circ - 90^\circ - 42^\circ = 48^\circ$$

$$\frac{b}{a} = \tan(\beta) \Rightarrow b = a \cdot \tan(\beta) = 11 \cdot \tan(42^\circ) \approx 9.904 \text{ m}$$

$$c = \sqrt{a^2 + b^2} \approx 14.8 \text{ m}$$

Aufgabe 3.2

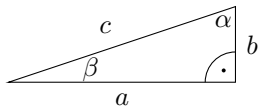


$$\gamma = 90^\circ - \alpha = 59^\circ$$

$$\frac{c}{b} = \cos(\alpha) \Rightarrow c = b \cdot \cos(\alpha) = 95 \cdot \cos(31^\circ) \approx 81.43 \text{ mm}$$

$$\frac{a}{b} = \sin(\alpha) \Rightarrow a = b \cdot \sin(\alpha) = 95 \cdot \sin(31^\circ) \approx 48.93 \text{ mm}$$

Aufgabe 3.3



$$b = \sqrt{c^2 - a^2} = \sqrt{176} = 13.27 \text{ cm}$$

$$\sin(\alpha) = \frac{a}{c} \Rightarrow \alpha = \arcsin\left(\frac{a}{c}\right) = \arcsin\left(\frac{7}{15}\right) \approx 27.82^\circ$$

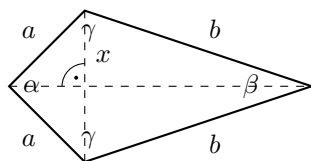
$$\beta = 90^\circ - \alpha \approx 62.18^\circ$$

Aufgabe 3.4

$$\frac{h_a}{b} = \sin(\alpha) \Rightarrow h_a = b \cdot \sin(\alpha) = 5 \cdot \sin(44^\circ) = 3.473 \text{ cm}$$

$$A = a \cdot h_a \approx 27.79 \text{ cm}^2$$

Aufgabe 3.5

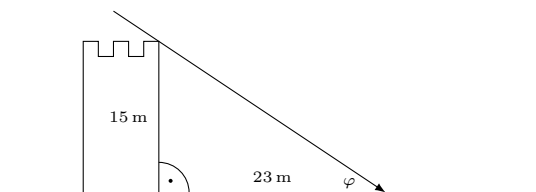


$$\frac{x}{a} = \sin\left(\frac{\alpha}{2}\right) \Rightarrow x = a \cdot \sin\left(\frac{\alpha}{2}\right) = 4 \cdot \sin(38^\circ) \approx 2.462 \text{ cm} \xrightarrow{\text{sto}} \mathbf{X}$$

$$\sin\left(\frac{\beta}{2}\right) = \frac{x}{b} \Rightarrow \frac{\beta}{2} = \arcsin\left(\frac{x}{b}\right) \Rightarrow \beta = 2 \cdot \arcsin\left(\frac{x}{b}\right) \approx 31.76^\circ$$

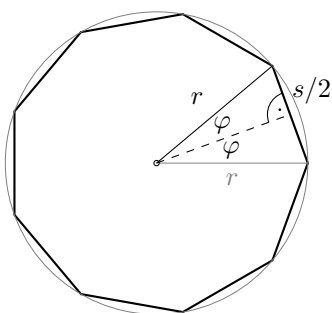
$$\gamma = (360^\circ - \alpha - \beta)/2 \approx 126.12^\circ$$

Aufgabe 3.6



$$\tan(\varphi) = \frac{15}{23} \Rightarrow \varphi = \arctan\left(\frac{15}{23}\right) \approx 33.11^\circ$$

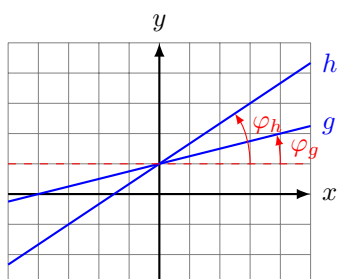
Aufgabe 3.7



$$s = 2 \cdot r \cdot \sin(\varphi) = 8 \cdot \sin(20^\circ) \approx 2.736 \text{ cm}$$

$$u = 9 \cdot s \approx 24.625 \text{ cm}$$

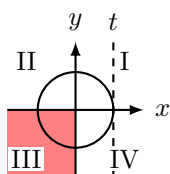
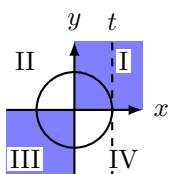
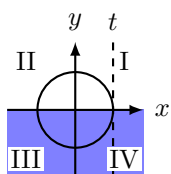
Aufgabe 3.8



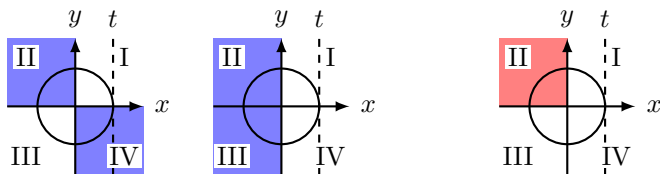
$$\Delta\varphi = \varphi_h - \varphi_g = \arctan\left(\frac{2}{3}\right) - \arctan\left(\frac{1}{4}\right) \approx 19.65^\circ$$

Aufgabe 4.1

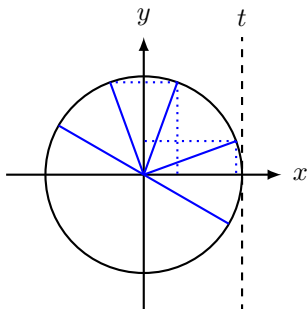
(a) $\sin(\varphi) < 0$ $\tan(\varphi) > 0$ $\sin(\varphi) < 0 \wedge \tan(\varphi) > 0$



(b) $\tan(\varphi) < 0$ $\cos(\varphi) < 0$ $\tan(\varphi) < 0 \wedge \cos(\varphi) < 0$



Aufgabe 4.2



$$\sin(20^\circ) = \cos(70^\circ)$$

$$\cos(750^\circ) = \cos(750 - 2 \cdot 360^\circ) = \cos(30^\circ)$$

$$\tan(330^\circ) = \tan(330^\circ - 180^\circ) = \tan(150^\circ)$$

$$\sin(70^\circ) = \sin(110^\circ)$$

Aufgabe 4.3

Multiplikationstabelle:

	$\sin \alpha$	$\cos \alpha$	1
$\sin \alpha$	$\sin^2 \alpha$	$\sin \alpha \cos \alpha$	$\sin \alpha$
$\cos \alpha$	$\sin \alpha \cos \alpha$	$\cos^2 \alpha$	$\cos \alpha$
-1	$-\sin \alpha$	$-\cos \alpha$	-1

$$(\sin \alpha + \cos \alpha + 1)(\sin \alpha + \cos \alpha - 1)$$

$$= \sin^2 \alpha + \cos^2 \alpha - 1 + 2 \sin \alpha \cos \alpha$$

$$= 1 - 1 + 2 \sin \alpha \cos \alpha = \sin 2\alpha$$

Aufgabe 4.4

$$\begin{aligned} \tan(180^\circ + \varphi) &= \frac{\sin(180^\circ + \varphi)}{\cos(180^\circ + \varphi)} = \frac{-\sin(\varphi)}{-\cos(\varphi)} \\ &= \frac{\sin(\varphi)}{\cos(\varphi)} = \tan(\varphi) \end{aligned}$$

Aufgabe 4.5

$$\begin{aligned}\cos(15^\circ) &= \cos(45^\circ - 30^\circ) \\ &= \cos(45^\circ)\cos(30^\circ) + \sin(45^\circ)\sin(30^\circ)\end{aligned}$$

$$\begin{aligned}\text{exakte Werte: } \cos(45^\circ) &= \sqrt{2}/2 & \sin(45^\circ) &= \sqrt{2}/2 \\ \cos(30^\circ) &= \sqrt{3}/2 & \sin(30^\circ) &= 1/2\end{aligned}$$

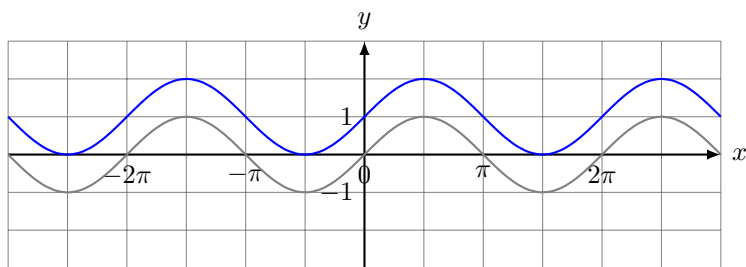
$$\cos(15^\circ) = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

Aufgabe 4.6

$$\begin{aligned}\cos 2\varphi &= \cos(\varphi + \varphi) = \cos \varphi \cos \varphi - \sin \varphi \sin \varphi \\ &= \cos^2 \varphi - \sin^2 \varphi\end{aligned}$$

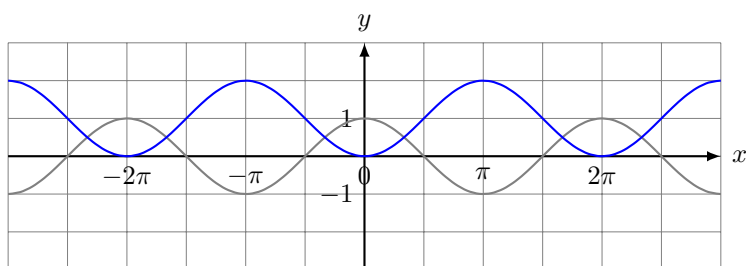
Aufgabe 5.1

$$y = \sin(x) + 1$$



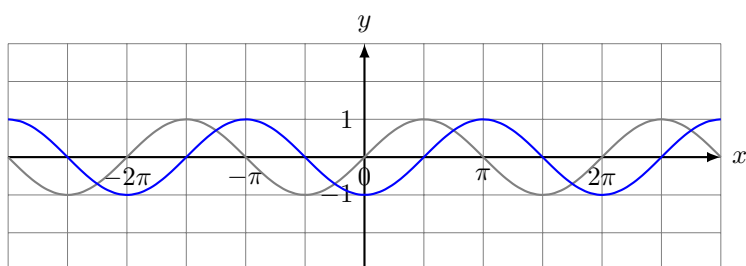
Aufgabe 5.2

$$y = 1 - \cos(x)$$



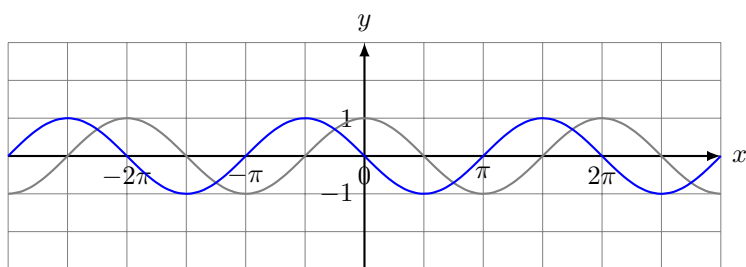
Aufgabe 5.3

$$y = \sin\left(x - \frac{\pi}{2}\right)$$



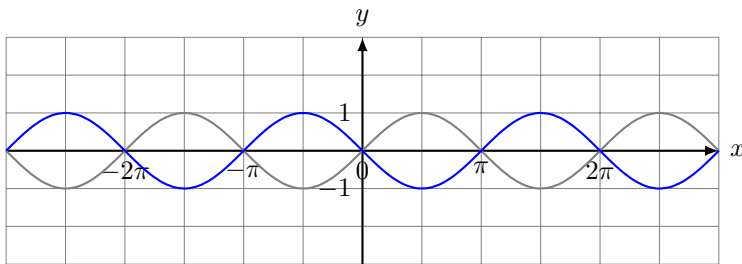
Aufgabe 5.4

$$y = \cos\left(x + \frac{\pi}{2}\right)$$



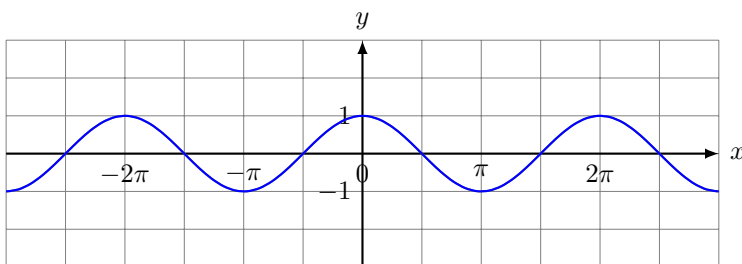
Aufgabe 5.5

$$y = \sin(-x)$$



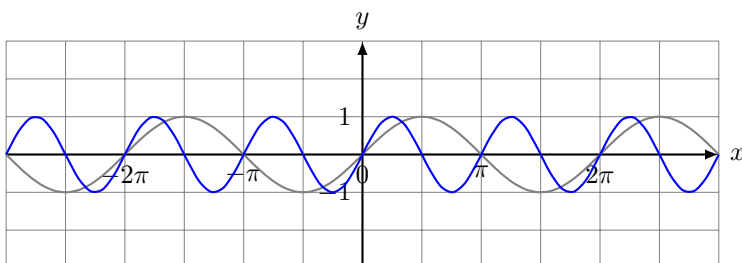
Aufgabe 5.6

$$y = \cos(-x)$$



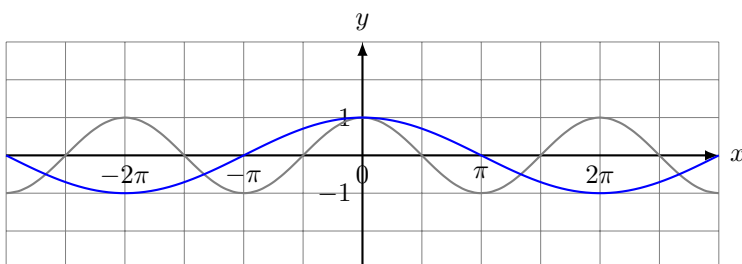
Aufgabe 5.7

$$y = \sin(2x)$$



Aufgabe 5.8

$$y = \cos\left(\frac{1}{2}x\right)$$



Aufgabe 6.1

$$\beta = 180^\circ - \alpha - \gamma$$

$$\beta = 80^\circ$$

$$\frac{a}{\sin \alpha} = \frac{c}{\sin \gamma} \Rightarrow a = \frac{c \cdot \sin \alpha}{\sin \gamma}$$

$$a = 4.45$$

$$b = \frac{c \cdot \sin \beta}{\sin \gamma}$$

$$b = 6.82$$

Aufgabe 6.2

$$\beta = 180^\circ - \alpha - \gamma$$

$$\beta = 76.3^\circ$$

$$\frac{c}{\sin \gamma} = \frac{b}{\sin \beta} \Rightarrow c = \frac{b \cdot \sin \gamma}{\sin \beta}$$

$$c = 3.17$$

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} \Rightarrow a = \frac{b \cdot \sin \alpha}{\sin \beta}$$

$$a = 5.11$$

Aufgabe 6.3

$$\gamma = 180^\circ - \alpha - \beta$$

$$\gamma = 81.2^\circ$$

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} \Rightarrow a = \frac{b \cdot \sin \alpha}{\sin \beta}$$

$$a = 4.82$$

$$\frac{c}{\sin \gamma} = \frac{b}{\sin \beta} \Rightarrow c = \frac{b \cdot \sin \gamma}{\sin \beta}$$

$$c = 4.94$$

Aufgabe 6.4

$$\frac{\sin \gamma}{c} = \frac{\sin \beta}{b} \Rightarrow \sin \gamma = \frac{c \cdot \sin \beta}{b}$$

$$\gamma = \arcsin \frac{c \cdot \sin \beta}{b}$$

$$\gamma = 48.74^\circ$$

$$\alpha = 180^\circ - \beta - \gamma$$

$$\alpha = 61.26^\circ$$

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} \Rightarrow a = \frac{b \cdot \sin \alpha}{\sin \beta}$$

$$a = 4.67$$

Aufgabe 6.5

$$\frac{\sin \gamma}{c} = \frac{\sin \beta}{b} \Rightarrow \sin \gamma = \frac{c \cdot \sin \beta}{b}$$

$$\gamma = \arcsin \frac{c \cdot \sin \beta}{b} \qquad \gamma = 10.89^\circ$$

$$\alpha = 180^\circ - \beta - \gamma \qquad \alpha = 144.91^\circ$$

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} \Rightarrow a = \frac{b \cdot \sin \alpha}{\sin \beta} \qquad a = 6.24$$

Aufgabe 6.6

Vorsicht: β ist der Gegenwinkel der kürzeren Seite b . Also gibt es zwei Lösungen.

$$\frac{\sin \gamma_1}{c} = \frac{\sin \beta}{b} \Rightarrow \dots \Rightarrow \gamma_1 = \arcsin \frac{c \cdot \sin \beta}{b} \qquad \gamma_1 = 69.84^\circ$$

$$\gamma_2 = 180^\circ - \gamma_1 \qquad \gamma_2 = 110.16^\circ$$

$$\alpha_1 = 180^\circ - \beta - \gamma_1 \qquad \alpha_1 = 73.76^\circ$$

$$\alpha_2 = 180^\circ - \beta - \gamma_2 \qquad \alpha_2 = 33.44^\circ$$

$$\frac{a_1}{\sin \alpha_1} = \frac{b}{\sin \beta} \Rightarrow a_1 = \frac{b \cdot \sin \alpha_1}{\sin \beta} \qquad a_1 = 37.48$$

$$\frac{a_2}{\sin \alpha_2} = \frac{b}{\sin \beta} \Rightarrow a_2 = \frac{b \cdot \sin \alpha_2}{\sin \beta} \qquad a_2 = 21.54$$

Aufgabe 6.7

Cosinussatz: $a^2 = b^2 + c^2 - 2bc \cos \alpha$

$$\cos \alpha = \frac{b^2 + c^2 - a^2}{2bc} \Rightarrow \alpha = \arccos \frac{b^2 + c^2 - a^2}{2bc} \qquad \alpha = 59.19^\circ$$

$$\cos \beta = \frac{c^2 + a^2 - b^2}{2ca} \Rightarrow \beta = \arccos \frac{c^2 + a^2 - b^2}{2ca} \qquad \beta = 39.39^\circ$$

$$\cos \gamma = \frac{a^2 + b^2 - c^2}{2ab} \Rightarrow \gamma = \arccos \frac{a^2 + b^2 - c^2}{2ab} \qquad \gamma = 81.42^\circ$$

Aufgabe 6.8

$$\alpha = \arccos \frac{b^2 + c^2 - a^2}{2bc} \qquad \alpha = 93.76^\circ$$

$$\beta = \arccos \frac{c^2 + a^2 - b^2}{2ca} \qquad \beta = 36.58^\circ$$

$$\gamma = \arccos \frac{a^2 + b^2 - c^2}{2ab} \qquad \gamma = 49.66^\circ$$

Aufgabe 6.9

$$\alpha = \arccos \frac{b^2 + c^2 - a^2}{2bc} \qquad \alpha = 131.41^\circ$$

$$\beta = \arccos \frac{c^2 + a^2 - b^2}{2ca} \qquad \beta = 15.19^\circ$$

$$\gamma = \arccos \frac{a^2 + b^2 - c^2}{2ab} \qquad \gamma = 33.40^\circ$$

Aufgabe 6.10

Cosinussatz: $c^2 = a^2 + b^2 - 2ab \cos \gamma$

$$c = \sqrt{a^2 + b^2 - 2ab \cos \gamma} \qquad c = 5.47$$

Sinussatz:

$$\frac{\sin \alpha}{a} = \frac{\sin \gamma}{c} \Rightarrow \alpha = \arcsin \frac{a \cdot \sin \gamma}{c} \qquad \alpha = 34.25^\circ$$

$$\beta = 180^\circ - \alpha - \gamma \qquad \beta = 41.45^\circ$$

Aufgabe 6.11

$$a = \sqrt{b^2 + c^2 - 2bc \cos \alpha} \qquad a = 27.70$$

$$\frac{\sin \beta}{b} = \frac{\sin \alpha}{a} \Rightarrow \beta = \arcsin \frac{b \cdot \sin \alpha}{a} \qquad \beta = 65.47^\circ$$

$$\gamma = 180^\circ - \alpha - \beta \qquad \gamma = 72.03^\circ$$

Aufgabe 6.12

$$a = \sqrt{b^2 + c^2 - 2bc \cos \alpha} \qquad a = 510.00$$

$$\frac{\sin \beta}{b} = \frac{\sin \alpha}{a} \Rightarrow \beta = \arcsin \frac{b \cdot \sin \alpha}{a} \qquad \beta = 17.06^\circ$$

$$\gamma = 180^\circ - \alpha - \beta \qquad \gamma = 45.24^\circ$$