

Aufgabe 1.1

- (a) $\log_2 8 = \log_2 2^3 = 3$
- (b) $\log_{10} 1\,000\,000 = \log_{10} 10^6 = 6$
- (c) $\log_{13} 169 = \log_{13} 13^2 = 2$
- (d) $\log_7 1 = \log_7 7^0 = 0$
- (e) $\log_6 216 = \log_6 216 = 3$
- (f) $\log_2 1024 = \log_2 2^{10} = 10$
- (g) $\log_9 9 = \log_9 9^1 = 1$
- (h) $\log_5 625 = \log_5 5^4 = 4$
- (i) $\log_4 64 = \log_4 4^3 = 3$
- (j) $\log_{\sqrt{3}} 3 = \log_{\sqrt{3}} \sqrt{3}^2 = 2$
- (k) $\log_{\sqrt{2}} 8 = \log_{\sqrt{2}} 2^3 = \log_{\sqrt{2}} ((\sqrt{2})^2)^3 = \log_{\sqrt{2}} (\sqrt{2})^6 = 6$
- (l) $\log_{\sqrt{5}} 5\sqrt{5} = \log_{\sqrt{5}} (\sqrt{5} \cdot \sqrt{5} \cdot \sqrt{5}) = \log_{\sqrt{5}} (\sqrt{5})^3 = 3$

Aufgabe 1.2

- (a) $\log_5 \frac{1}{25} = \log_5 \frac{1}{5^2} = \log_5 5^{-2} = -2$
- (b) $\log_2 \frac{1}{128} = \log_2 \frac{1}{2^7} = \log_2 2^{-7} = -7$
- (c) $\log_3 \frac{1}{81} = \log_3 \frac{1}{3^4} = \log_3 3^{-4} = -4$
- (d) $\log_{10} 0.001 = \log_{10} \frac{1}{1000} = \log_{10} \frac{1}{10^3} = \log_{10} 10^{-3} = -3$
- (e) $\log_8 0.125 = \log_8 \frac{1}{8} = \log_8 8^{-1} = -1$
- (f) $\log_{\frac{1}{3}} 9 = \log_{\frac{1}{3}} 3^2 = \log_{\frac{1}{3}} \left(\frac{1}{3}\right)^{-2} = -2$
- (g) $\log_{\frac{3}{7}} \frac{7}{3} = \log_{\frac{3}{7}} \left(\frac{3}{7}\right)^{-1} = -1$
- (h) $\log_{\frac{2}{3}} \frac{27}{8} = \log_{\frac{2}{3}} \frac{3^3}{2^3} = \log_{\frac{2}{3}} \left(\frac{3}{2}\right)^3 = \log_{\frac{2}{3}} \left(\frac{2}{3}\right)^{-3} = -3$
- (i) $\log_2 0.0625 = \log_2 \frac{625}{10000} = \log_2 \frac{5^4}{10^4} = \log_2 \left(\frac{5}{10}\right)^4 = \log_2 \left(\frac{1}{2}\right)^4 = \log_2 2^{-4} = -4$

Aufgabe 1.3

(a) Lösungsweg 1 (als Exponentialgleichung darstellen):

$$\log_4 2 = x \Rightarrow 4^x = 2$$

$$2^{2x} = 2^1$$

$$2x = 1$$

$$x = \frac{1}{2}$$

Lösungsweg 2 (Numerus umformen):

$$\log_4 2 = \log_4 4^{\frac{1}{2}} = \frac{1}{2}$$

Aus Platzgründen wird jeweils nur Lösungsweg 2 dargestellt.

(b) $\log_{27} 3 = \log_{27} 27^{\frac{1}{3}} = \frac{1}{3}$

(c) $\log_3 \sqrt{3} = \log_3 3^{\frac{1}{2}} = \frac{1}{2}$

(d) $\log_{16} 2 = \log_{16} 16^{\frac{1}{4}} = \frac{1}{4}$

(e) $\log_{1000} 10 = \log_{1000} 1000^{\frac{1}{3}} = \frac{1}{3}$

(f) $\log_2 \sqrt[5]{2} = \log_2 2^{\frac{1}{5}} = \frac{1}{5}$

(g) $\log_{512} 2 = \log_{512} 512^{\frac{1}{9}} = \frac{1}{9}$

(h) $\log_{216} 6 = \log_{216} 216^{\frac{1}{3}} = \frac{1}{3}$

(i) $\log_{16} \sqrt{2} = \log_{16} 16^{\frac{1}{8}} = \frac{1}{8}$

Aufgabe 1.4

(a) $\log_9 27 = x \Rightarrow 9^x = 27$

$$3^{2x} = 3^3$$

$$2x = 3$$

$$x = \frac{3}{2}$$

(b) $\log_{32} 8 = x \Rightarrow 32^x = 8$

$$2^{5x} = 2^3$$

$$5x = 3$$

$$x = \frac{3}{5}$$

(c) $\log_{1000} 100 = x \Rightarrow 1000^x = 100$

$$10^{3x} = 10^2$$

$$3x = 2$$

$$x = \frac{2}{3}$$

$$\begin{aligned}
 \text{(d) } \log_8 16 = x &\Rightarrow 8^x = 16 \\
 &2^{3x} = 2^4 \\
 &3x = 4 \\
 &x = \frac{4}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{(e) } \log_{243} 27 = x &\Rightarrow 243^x = 27 \\
 &3^{5x} = 3^3 \\
 &5x = 3 \\
 &x = \frac{3}{5}
 \end{aligned}$$

$$\begin{aligned}
 \text{(f) } \log_{125} 25 = x &\Rightarrow 125^x = 25 \\
 &5^{3x} = 5^2 \\
 &3x = 2 \\
 &x = \frac{2}{3}
 \end{aligned}$$

Aufgabe 1.5

$$\text{(a) } \log_3 3^{\frac{4}{7}} = \frac{4}{7}$$

$$\text{(b) } \log_2 4^{-9} = \log_2 (2^2)^{-9} = \log_2 2^{-18} = -18$$

$$\begin{aligned}
 \text{(c) } \log_{16} 8^7 = x &\Rightarrow 16^x = 8^7 \\
 &2^{4x} = 2^{21} \\
 &4x = 21 \\
 &x = \frac{21}{4}
 \end{aligned}$$

Aufgabe 1.6

$$\begin{aligned}
 \text{(a) } \log_9 \frac{1}{27} = x &\Rightarrow 9^x = \frac{1}{27} \\
 &(3^2)^x = \frac{1}{3^3} \\
 &3^{2x} = 3^{-3} \\
 &2x = -3 \\
 &x = -\frac{3}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b) } \log_{0.125} 4 = x &\Rightarrow 0.125^x = 4 \\
 &\left(\frac{1}{8}\right)^x = 2^2 \\
 &2^{-3x} = 2^2 \\
 &-3x = 2 \\
 &x = -\frac{2}{3}
 \end{aligned}$$

$$\begin{aligned}
\text{(c) } \log_{2\sqrt{2}} \frac{1}{4} = x &\Rightarrow (2\sqrt{2})^x = \frac{1}{4} \\
&(2^1 \cdot 2^{\frac{1}{2}})^x = 2^{-2} \\
&(2^{\frac{3}{2}})^x = 2^{-2} \\
&2^{\frac{3}{2}x} = 2^{-2} \\
&\frac{3}{2}x = -2 \\
&x = -\frac{4}{3}
\end{aligned}$$

Aufgabe 1.7

- (a) $\log_a a^4 = 4$
- (b) $\log_a a = \log_a a^1 = 1$
- (c) $\log_a \frac{1}{a^2} = \log_a a^{-2} = -2$
- (d) $\log_a a^{-\frac{3}{4}} = -\frac{3}{4}$
- (e) $\log_a 1 = \log_a a^0 = 0$
- (f) $\log_a (a^2)^3 = \log_a a^6 = 6$
- (g) $\log_a \sqrt{a} = \log_a a^{\frac{1}{2}} = \frac{1}{2}$
- (h) $\log_a \sqrt[3]{a^5} = \log_a a^{\frac{5}{3}} = \frac{5}{3}$
- (i) $\log_a a\sqrt{a} = \log_a (a^1 \cdot a^{\frac{1}{2}}) = \log_a a^{\frac{3}{2}} = \frac{3}{2}$

Aufgabe 1.8

- (a) $x^3 = 27 = 3^3 \Rightarrow x = 3$
- (b) $x^{-9} = 512 = 2^9 = \left(\frac{1}{2}\right)^{-9} \Rightarrow x = \frac{1}{2}$
- (c) $x^3 = 10^6 = (10^2)^3 \Rightarrow x = 100$
- (d) $x^{-\frac{1}{2}} = 4 = \sqrt{16} = 16^{\frac{1}{2}} = \left(\frac{1}{16}\right)^{-\frac{1}{2}} \Rightarrow x = \frac{1}{16}$
- (e) $x^4 = 4 = 2^2 = \sqrt{2^4} \Rightarrow x = \sqrt{2}$
- (f) $x^0 = 1 \neq 3 \Rightarrow$ keine Lösung

Aufgabe 1.9

- (a) $x = 5^3 = 125$
- (b) $x = 4^1 = 4$
- (c) $x = (\sqrt{3})^6 = (3^{\frac{1}{2}})^6 = 3^3 = 27$

$$(d) x = 49^{-\frac{1}{2}} = (7^2)^{-\frac{1}{2}} = 7^{-1} = \frac{1}{7}$$

$$(e) x = 4^{1.5} = (2^2)^{1.5} = 2^{2 \cdot 1.5} = 2^3 = 8$$

$$(f) x = 3^0 = 1$$

Aufgabe 1.10

$$(a) \log_{10} (\log_{10} 10^{100}) = \log_{10} 100 = 2$$

$$(b) \log_3 (\log_2 8) = \log_3 3 = 1$$

$$(c) \log_6 (\log_5 5) = \log_6 1 = 0$$

Aufgabe 1.11

$$(a) 2^{\log_2 32} = 2^5 = 32$$

$$(b) 3^{\log_3 7} = 7$$

$$(c) 4^{\log_2 5} = (2^2)^{\log_2 5} = (2^{\log_2 5})^2 = 5^2 = 25$$

Aufgabe 2.1

$$(a) \log_a (bc) = \log_a b + \log_a c$$

$$(b) \log_a (apq) = 1 + \log_a p + \log_a q$$

$$(c) \log_a c^3 = 3 \log_a c$$

$$(d) \log_a \sqrt{s} = \frac{1}{2} \log_a s$$

$$(e) \log_a (1/m) = -\log_a m$$

$$(f) \log_a (x/y) = \log_a x - \log_a y$$

$$(g) \log_a \frac{x}{ay} = \log_a x - 1 - \log_a y$$

$$(h) \log_a (a^4 b^5) = 4 + 5 \log_a b$$

$$(i) \log_a \frac{z}{x+y} = \log_a z - \log_a (x+y)$$

$$(j) \log_a \sqrt[3]{b} = \frac{1}{3} \log_a b$$

$$(k) \log_a \frac{xy^2}{2v^3} = \log_a x + 2 \log_a y - \log_a 2 - 3 \log_a v$$

$$(l) \log_a \sqrt{b^5 c^3} = \frac{5}{2} \log_a b + \frac{3}{2} \log_a c$$

Aufgabe 2.2

- (a) $\log_a p + \log_a q = \log_a(pq)$
- (b) $\log_a e - \log_a f = \log_a(e/f)$
- (c) $3 \log_a x = \log_a x^3$
- (d) $\frac{1}{2} \log_a y = \log_a \sqrt{y}$
- (e) $1 + \log_a c = \log_a(ac)$
- (f) $2 \log_a b + 3 \log_a c - 5 \log_a d = \log_a(b^2 c^3 / d^5)$
- (g) $-\log_a x - \log_a y - \log_a z = \log_a \frac{1}{xyz}$
- (h) $\frac{1}{3}(\log_a s + 2 \log_a t) = \log_a (s^{\frac{1}{3}} t^{\frac{2}{3}})$
- (i) $\log_a c^3 + \log_a c^5 - \log_a \sqrt{c} = \log_a c^{7.5}$
- (j) $m \log_a y - k \log_a \frac{1}{z} = \log_a (y^m z^k)$

Aufgabe 3.1

- (a) $\frac{\log_{10} \pi}{\log_{10} 5} = \log_5 \pi$
- (b) $\frac{\ln 60}{\ln 7} = \log_7 60$
- (c) $\frac{\log_3 \sqrt{2}}{\log_3 9} = \log_9 \sqrt{2}$

Aufgabe 3.2

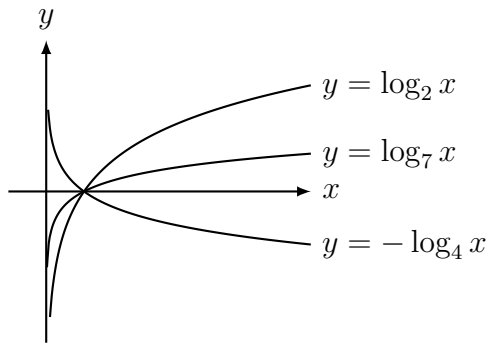
- (a) $\log_3 5 \cdot \log_5 7 \cdot \log_7 9 = \frac{\log_{10} 5}{\log_{10} 3} \cdot \frac{\log_{10} 7}{\log_{10} 5} \cdot \frac{\log_{10} 9}{\log_{10} 7} = \frac{\log_{10} 9}{\log_{10} 3} = \log_3 9 = 2$
- (b)
$$\begin{aligned} \frac{\log_3 13 \cdot \log_5 17}{\log_3 289 \cdot \log_5 169} &= \frac{\frac{\log_{10} 13}{\log_{10} 3} \cdot \frac{\log_{10} 17}{\log_{10} 5}}{\frac{\log_{10} 289}{\log_{10} 3} \cdot \frac{\log_{10} 169}{\log_{10} 5}} \\ &= \frac{\log_{10} 13}{\log_{10} 3} \cdot \frac{\log_{10} 17}{\log_{10} 5} \cdot \frac{\log_{10} 3}{\log_{10} 289} \cdot \frac{\log_{10} 5}{\log_{10} 169} \\ &= \frac{\log_{10} 13}{\log_{10} 3} \cdot \frac{\log_{10} 17}{\log_{10} 5} \cdot \frac{\log_{10} 3}{2 \log_{10} 17} \cdot \frac{\log_{10} 5}{2 \log_{10} 13} = \frac{1}{4} \end{aligned}$$

Aufgabe 3.3

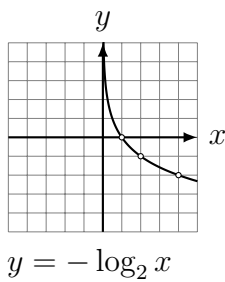
$$(a) \log_a b \cdot \log_b a = \log_a b \cdot \frac{\log_a a}{\log_a b} = \log_a a = 1$$

$$(b) \log_c b : \log_a b = \log_c b : \frac{\log_c b}{\log_c a} = \log_c b \cdot \frac{\log_c a}{\log_c b} = \log_c a$$

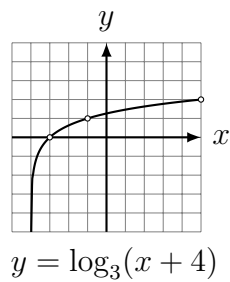
Aufgabe 4.1



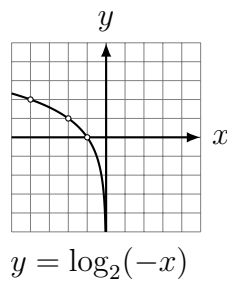
Aufgabe 4.2



$$y = -\log_2 x$$



$$y = \log_3(x + 4)$$



$$y = \log_2(-x)$$

Aufgabe 4.10

$$\log_2 x + \log_2 3 = 2$$

$$\log_2(x \cdot 3) = \log_2 4$$

$$3x = 4$$

$$x = \frac{4}{3} \quad (\text{Probe: ok})$$

$$L = \left\{ \frac{4}{3} \right\}$$

Aufgabe 4.11

$$\begin{aligned} \log_3(x-2) - \log_3(x+3) &= \log_3 2 \\ \log_3 \frac{x-2}{x+3} &= \log_3 2 \\ \frac{x-2}{x+3} &= 2 \\ x-2 &= 2x+6 \\ -8 &= x \quad (\text{Probe: } \log_3(-10) - \dots \text{ verboten}) \\ L &= \{\} \end{aligned}$$

Aufgabe 4.12

$$\begin{aligned} \log_4 x + \log_2 5 &= 1 \\ \log_4 x + \log_4 25 &= \log_4 4 \\ \log_4(25x) &= \log_4 4 \\ x &= \frac{4}{25} \quad (\text{Probe: ok}) \\ L &= \left\{ \frac{4}{25} \right\} \end{aligned}$$

Aufgabe 4.13

$$\begin{aligned} \log_5 x^2 + \log_5 2 &= \log_5(x+6) \\ \log_5(2x^2) &= \log_5(x+6) \\ 2x^2 &= x+6 \\ 2x^2 - x - 6 &= 0 \quad (a=2, b=-1, c=-6) \\ D &= b^2 - 4ac = (-1)^2 - 4 \cdot 2 \cdot (-6) = 49 \\ x_1 &= \frac{-b + \sqrt{D}}{2a} = \frac{1+7}{4} = 2 \quad (\text{Probe: ok}) \\ x_2 &= \frac{-b - \sqrt{D}}{2a} = \frac{1-7}{4} = -1.5 \quad (\text{Probe: ok}) \\ L &= \{-1.5, 2\} \end{aligned}$$