Logarithmen

Übungen (L+)

Version vom 16. Februar 2022

Aufgabe 1.1

(a) $\log_2 8 = \log_2 2^3 = 3$

(b) $\log_{10} 1\,000\,000 = \log_{10} 10^6 = 6$

(c) $\log_{13} 169 = \log_{13} 13^2 = 2$

(d) $\log_7 1 = \log_7 7^0 = 0$

(e) $\log_6 216 = \log_6 216 = 3$

(f) $\log_2 1024 = \log_2 2^{10} = 10$

(g) $\log_9 9 = \log_9 9^1 = 1$

(h) $\log_5 625 = \log_5 5^4 = 4$

(i) $\log_4 64 = \log_4 4^3 = 3$

(j) $\log_{\sqrt{3}} 3 = \log_{\sqrt{3}} \sqrt{3}^2 = 2$

(k) $\log_{\sqrt{2}} 8 = \log_{\sqrt{2}} 2^3 = \log_{\sqrt{2}} \left(\left(\sqrt{2}\right)^2\right)^3 = \log_{\sqrt{2}} \left(\sqrt{2}\right)^6 = 6$

(l) $\log_{\sqrt{5}} 5\sqrt{5} = \log_{\sqrt{5}} \left(\sqrt{5} \cdot \sqrt{5} \cdot \sqrt{5}\right) = \log_{\sqrt{5}} \left(\sqrt{5}\right)^3 = 3$

Aufgabe 1.2

(a) $\log_5 \frac{1}{25} = \log_5 \frac{1}{5^2} = \log_5 5^{-2} = -2$

(b) $\log_2 \frac{1}{128} = \log_2 \frac{1}{2^7} = \log_2 2^{-7} = -7$

(c) $\log_3 \frac{1}{81} = \log_3 \frac{1}{3^4} = \log_3 3^{-4} = -4$

(d) $\log_{10} 0.001 = \log_{10} \frac{1}{1000} = \log_{10} \frac{1}{10^3} = \log_{10} 10^{-3} = -3$

(e) $\log_8 0.125 = \log_8 \frac{1}{8} = \log_8 8^{-1} = -1$

(f) $\log_{\frac{1}{3}} 9 = \log_{\frac{1}{3}} 3^2 = \log_{\frac{1}{3}} \left(\frac{1}{3}\right)^{-2} = -2$

(g) $\log_{\frac{3}{7}} \frac{7}{3} = \log_{\frac{3}{7}} \left(\frac{3}{7}\right)^{-1} = -1$

(h) $\log_{\frac{2}{3}} \frac{27}{8} = \log_{\frac{2}{3}} \frac{3^3}{2^3} = \log_{\frac{2}{3}} \left(\frac{3}{2}\right)^3 = \log_{\frac{2}{3}} \left(\frac{2}{3}\right)^{-3} = -3$

(i) $\log_2 0.0625 = \log_2 \frac{625}{10\,000} = \log_2 \frac{5^4}{10^4} = \log_2 \left(\frac{5}{10}\right)^4$
$= \log_2 \left(\frac{1}{2}\right)^4 = \log_2 2^{-4} = -4$

Aufgabe 1.3

(a) $\log_4 2 = x \;\Rightarrow\; 4^x = 2 \;\Rightarrow\; 2^{2x} = 2^1 \;\Rightarrow\; 2x = 1 \;\Rightarrow\; x = \frac{1}{2}$

oder: $\log_4 2 = \log_4 4^{\frac{1}{2}} = \frac{1}{2}$

(b) $\log_{27} 3 = \log_{27} 27^{\frac{1}{3}} = \frac{1}{3}$

(c) $\log_3 \sqrt{3} = \log_3 3^{\frac{1}{2}} = \frac{1}{2}$

(d) $\log_{16} 2 = \log_{16} 16^{\frac{1}{4}} = \frac{1}{4}$

(e) $\log_{1000} 10 = \log_{1000} = \log_{1000} 1000^{\frac{1}{3}} = \frac{1}{3}$

(f) $\log_2 \sqrt[5]{2} = \log_2 2^{\frac{1}{5}} = \frac{1}{5}$

(g) $\log_{512} 2 = \log_{512} 512^{\frac{1}{9}} = \frac{1}{9}$

(h) $\log_{216} 6 = \log_{216} 216^{\frac{1}{3}} = \frac{1}{3}$

(i) $\log_{16} \sqrt{2} = \log_{16} 16^{\frac{1}{8}} = \frac{1}{8}$

Aufgabe 1.4

(a) $\log_9 27 = x \quad \Rightarrow \quad 9^x = 27$
$3^{2x} = 3^3$
$2x = 3$
$x = \frac{3}{2}$

(b) $\log_{32} 8 = x \quad \Rightarrow \quad 32^x = 8$
$2^{5x} = 2^3$
$5x = 3$
$x = \frac{3}{5}$

(c) $\log_{1000} 100 = x \quad \Rightarrow \quad 1000^x = 100$
$10^{3x} = 10^2$
$3x = 2$
$x = \frac{2}{3}$

(d) $\log_8 16 = x \quad \Rightarrow \quad 8^x = 16$
$2^{3x} = 2^4$
$3x = 4$
$x = \frac{4}{3}$

(e) $\log_{243} 27 = x \quad \Rightarrow \quad 243^x = 27$

$$3^{5x} = 3^3$$
$$5x = 3$$
$$x = \tfrac{3}{5}$$

(f) $\log_{125} 25 = x \quad \Rightarrow \quad 125^x = 25$

$$5^{3x} = 5^2$$
$$3x = 2$$
$$x = \tfrac{2}{3}$$

Aufgabe 1.5

(a) $\log_3 3^{\frac{4}{7}} = \frac{4}{7}$

(b) $\log_2 4^{-9} = \log_2 (2^2)^{-9} = \log_2 2^{-18} = -18$

(c) $\log_{16} 8^7 = x \quad \Rightarrow \quad 16^x = 8^7$

$$2^{4x} = 2^{21}$$
$$4x = 21$$
$$x = \tfrac{21}{4}$$

Aufgabe 1.6

(a) $\log_9 \frac{1}{27} = x \quad \Rightarrow \quad 9^x = \frac{1}{27}$

$$(3^2)x = \tfrac{1}{3^3}$$
$$3^{2x} = 3^{-3}$$
$$2x = -3$$
$$x = -\tfrac{3}{2}$$

(b) $\log_{0.125} 4 = x \quad \Rightarrow \quad 0.125^x = 4$

$$\left(\tfrac{1}{8}\right)^x = 2^2$$
$$2^{-3x} = 2^2$$
$$-3x = 2$$
$$x = -\tfrac{2}{3}$$

(c) $\log_{2\sqrt{2}} \frac{1}{4} = x \quad \Rightarrow \quad \left(2\sqrt{2}\right)^x = \frac{1}{4}$

$$\left(2^1 \cdot 2^{\frac{1}{2}}\right)^x = 2^{-2}$$
$$\left(2^{\frac{3}{2}}\right)^x = 2^{-2}$$
$$2^{\frac{3}{2}x} = 2^{-2}$$
$$\tfrac{3}{2}x = -2$$
$$x = -\tfrac{4}{3}$$

Aufgabe 1.7

(a) $\log_a a^4 = 4$

(b) $\log_a a = \log_a a^1 = 1$

(c) $\log_a \frac{1}{a^2} = \log_a a^{-2} = -2$

(d) $\log_a a^{-\frac{3}{4}} = -\frac{3}{4}$

(e) $\log_a 1 = \log_a a^0 = 0$

(f) $\log_a (a^2)^3 = \log_a a^6 = 6$

(g) $\log_a \sqrt{a} = \log_a a^{\frac{1}{2}} = \frac{1}{2}$

(h) $\log_a \sqrt[3]{a^5} = \log_a a^{\frac{5}{3}} = \frac{5}{3}$

(i) $\log_a a\sqrt{a} = \log_a \left(a^1 \cdot a^{\frac{1}{2}}\right) = \log_a a^{\frac{3}{2}} = \frac{3}{2}$

Aufgabe 1.8

(a) $x^3 = 27 = 3^3 \quad \Rightarrow \quad x = 3$

(b) $x^{-9} = 512 = 2^9 = \left(\frac{1}{2}\right)^{-9} \quad \Rightarrow \quad x = \frac{1}{2}$

(c) $x^3 = 10^6 = (10^2)^3 \quad \Rightarrow \quad x = 100$

(d) $x^{-\frac{1}{2}} = 4 = \sqrt{16} = 16^{\frac{1}{2}} = \left(\frac{1}{16}\right)^{-\frac{1}{2}} \quad \Rightarrow \quad x = \frac{1}{16}$

(e) $x^4 = 4 = 2^2 = \sqrt{2}^4 \quad \Rightarrow \quad x = \sqrt{2}$

(f) $x^0 = 1 \neq 3 \quad \Rightarrow \quad$ keine Lösung

Aufgabe 1.9

(a) $x = 5^3 = 125$

(b) $x = 4^1 = 4$

(c) $x = (\sqrt{3})^6 = (3^{\frac{1}{2}})^6 = 3^3 = 27$

(d) $x = 49^{-\frac{1}{2}} = (7^2)^{-\frac{1}{2}} = 7^{-1} = \frac{1}{7}$

(e) $x = 4^{1.5} = (2^2)^{1.5} = 2^{2 \cdot 1.5} = 2^3 = 8$

(f) $x = 3^0 = 1$

Aufgabe 1.10

(a) $\log_{10}\left(\log_{10} 10^{100}\right) = \log_{10} 100 = 2$

(b) $\log_3\left(\log_2 8\right) = \log_3 3 = 1$

(c) $\log_6\left(\log_5 5\right) = \log_6 1 = 0$

Aufgabe 1.11

(a) $2^{\log_2 32} = 2^5 = 32$

(b) $3^{\log_3 7} = 7$

(c) $4^{\log_2 5} = \left(2^2\right)^{\log_2 5} = \left(2^{\log_2 5}\right)^2 = 5^2 = 25$

Aufgabe 2.1

(a) $\log_a(bc) = \log_a b + \log_a c$

(b) $\log_a(apq) = 1 + \log_a p + \log_a q$

(c) $\log_a c^3 = 3\log_a c$

(d) $\log_a \sqrt{s} = \frac{1}{2}\log_a s$

(e) $\log_a \dfrac{1}{m} = \log_a m^{-1} = -\log_a m$

(f) $\log_a \dfrac{x}{y} = \log_a x - \log_a y$

(g) $\log_a \dfrac{x}{ay} = \log_a x - 1 - \log_a y$

(h) $\log_a(a^4 b^5) = 4 + 5\log_a b$

(i) $\log_a \dfrac{z}{x+y} = \log_a z - \log_a(x+y)$

(j) $\log_a \sqrt[3]{b} = \frac{1}{3}\log_a b$

(k) $\log_a \dfrac{xy^2}{2v^3} = \log_a x + 2\log_a y - \log_a 2 - 3\log_a v$

(l) $\log_a \sqrt{b^5 c^3} = \frac{5}{2}\log_a b + \frac{3}{2}\log_a c$

Aufgabe 2.2

(a) $\log_a p + \log_a q = \log_a(pq)$

(b) $\log_a e - \log_a f = \log_a \dfrac{e}{f}$

(c) $3\log_a x = \log_a x^3$

(d) $\frac{1}{2}\log_a y = \log_a \sqrt{y}$

(e) $1 + \log_a c = \log_a(ac)$

(f) $2\log_a b + 3\log_a c - 5\log_a d = \log_a \dfrac{b^2c^3}{d^5}$

(g) $-\log_a x - \log_a y - \log_a z = \log_a \dfrac{1}{xyz}$

(h) $\frac{1}{3}\big(\log_a s + 2\log_a t\big) = \log_a \big(s^{\frac{1}{3}} t^{\frac{2}{3}}\big)$

(i) $\log_a c^3 + \log_a c^5 - \log_a \sqrt{c} = \log_a c^{7.5}$

(j) $m\log_a y - k\log_a \frac{1}{z} = \log_a(y^m z^k)$

Aufgabe 2.3

(a) $4^{756} = 10^{\lg(4^{756})} = 10^{756\cdot\lg 4} = 10^{455.157}$
$= 10^{0.157} \cdot 10^{455} = 1.435 \cdot 10^{455}$

erste Ziffer: 1

Anzahl Stellen: 456

$4^1 = 4$, $4^2 = 16$, $4^3 = 64$, $4^4 = 256$, ...

$\mathrm{mod}(756, 2) = 0 \quad \Rightarrow \quad$ letzte Ziffer: 6

(b) $2^{908} = 10^{\lg(2^{908})} = 10^{908\cdot\lg 2} = 10^{273.335}$
$= 10^{0.335} \cdot 10^{273} = 2.163 \cdot 10^{273}$

erste Ziffer: 2

Anzahl Stellen: 274

$2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$, ...,

$\mathrm{mod}(908, 4) = 0 \quad \Rightarrow \quad$ letzte Ziffer: 6

(c) $5^{642} = 10^{\lg(5^{642})} = 10^{642 \cdot \lg 5} = 10^{448.739}$

$= 10^{0.739} \cdot 10^{448} = 5.479 \cdot 10^{448}$

erste Ziffer: 5

Anzahl Stellen: 449

$5^1 = 5$, $5^2 = 25$, ...

letzte Ziffer: 5

Aufgabe 3.1

(a) 1.7712

(b) 0.56458

(c) -3.0213

(d) 10.057

(e) 0.092735

(f) 1.6528

Aufgabe 3.2

(a) $\dfrac{\log_{10} \pi}{\log_{10} 5} = \log_5 \pi$

(b) $\dfrac{\ln 60}{\ln 7} = \log_7 60$

(c) $\dfrac{\log_3 \sqrt{2}}{\log_3 9} = \log_9 \sqrt{2}$

Aufgabe 3.3

(a) $\log_3 5 \cdot \log_5 7 \cdot \log_7 9 = \dfrac{\log_{10} 5}{\log_{10} 3} \cdot \dfrac{\log_{10} 7}{\log_{10} 5} \cdot \dfrac{\log_{10} 9}{\log_{10} 7}$

$= \dfrac{\log_{10} 9}{\log_{10} 3} = \log_3 9 = 2$

(b) $$\frac{\log_3 13 \cdot \log_5 17}{\log_3 289 \cdot \log_5 169} = \frac{\frac{\log_{10} 13}{\log_{10} 3} \cdot \frac{\log_{10} 17}{\log_{10} 5}}{\frac{\log_{10} 289}{\log_{10} 3} \cdot \frac{\log_{10} 169}{\log_{10} 5}}$$

$$= \frac{\log_{10} 13}{\log_{10} 3} \cdot \frac{\log_{10} 17}{\log_{10} 5} \cdot \frac{\log_{10} 3}{\log_{10} 289} \cdot \frac{\log_{10} 5}{\log_{10} 169}$$

$$= \frac{\log_{10} 13}{\log_{10} 3} \cdot \frac{\log_{10} 17}{\log_{10} 5} \cdot \frac{\log_{10} 3}{2\log_{10} 17} \cdot \frac{\log_{10} 5}{2\log_{10} 13}$$

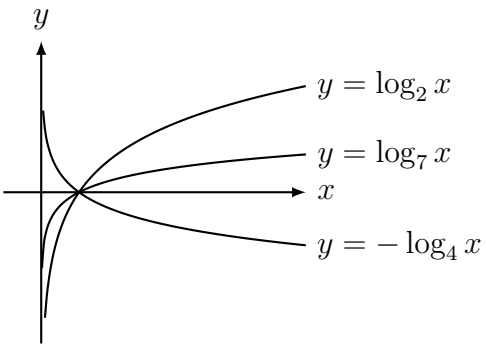
$$= \frac{1}{4}$$

Aufgabe 3.4

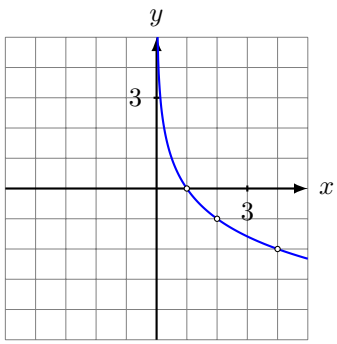
(a) $\log_a b \cdot \log_b a = \log_a b \cdot \frac{\log_a a}{\log_a b} = \log_a a = 1$

(b) $\log_c b : \log_a b = \log_c b : \frac{\log_c b}{\log_c a} = \log_c b \cdot \frac{\log_c a}{\log_c b} = \log_c a$

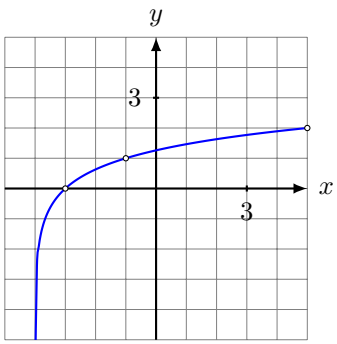
Aufgabe 4.1



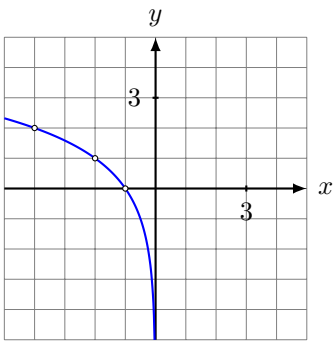
Aufgabe 4.2



$y = -\log_2 x$



$y = \log_3(x+4)$



$y = \log_2(-x)$

Aufgabe 5.1

$$\begin{aligned}
\log_2 x + \log_2 3 &= 2 \\
\log_2(x \cdot 3) &= \log_2 4 \\
3x &= 4 \\
x &= \tfrac{4}{3} \quad \text{(Probe: ok)} \\
L &= \{\tfrac{4}{3}\}
\end{aligned}$$

Aufgabe 5.2

$$\begin{aligned}
\log_3(x-2) - \log_3(x+3) &= \log_3 2 \\
\log_3 \frac{x-2}{x+3} &= \log_3 2 \\
\frac{x-2}{x+3} &= 2 \\
x - 2 &= 2x + 6 \\
-8 &= x \quad \text{(Probe: } \log_3(-10) - \ldots \text{ verboten)} \\
L &= \{\,\}
\end{aligned}$$

Aufgabe 5.3

$$\begin{aligned}
\log_4 x + \log_2 5 &= 1 \\
\log_4 x + \log_4 25 &= \log_4 4 \\
\log_4(25x) &= \log_4 4 \\
x &= \tfrac{4}{25} \quad \text{(Probe: ok)} \\
L &= \{\tfrac{4}{25}\}
\end{aligned}$$

Aufgabe 5.4

$$\begin{aligned}
\log_5 x^2 + \log_5 2 &= \log_5(x+6) \\
\log_5(2x^2) &= \log_5(x+6) \\
2x^2 &= x + 6 \\
2x^2 - x - 6 &= 0 \quad (a = 2,\, b = -1,\, c = -6) \\
D &= b^2 - 4ac = (-1)^2 - 4 \cdot 2 \cdot (-6) = 49 \\
x_1 &= \frac{-b + \sqrt{D}}{2a} = \frac{1+7}{4} = 2 \quad \text{(Probe: ok)} \\
x_2 &= \frac{-b - \sqrt{D}}{2a} = \frac{1-7}{4} = -1.5 \quad \text{(Probe: ok)} \\
L &= \{-1.5,\, 2\}
\end{aligned}$$

Aufgabe 5.5

$$\begin{aligned}
\log_{10} 10^x &= \log_{10} 2341 \\
x \cdot \log_{10} 10 &= \log_{10} 2341 \\
x &= 3.369
\end{aligned}$$

Aufgabe 5.6

$$\begin{aligned} 3^x &= 15 \\ \log_{10} 3^x &= \log_{10} 15 \\ x \cdot \log_{10} 3 &= \log_{10} 15 \\ x &= \frac{\log_{10} 15}{\log_{10} 3} \\ x &= 2.465 \end{aligned}$$

Aufgabe 5.7

$$\begin{aligned} 2^x &= \frac{13}{9} \\ \log_{10} 2^x &= \log_{10} \frac{13}{9} \\ x \cdot \log_{10} 2 &= \log_{10} \frac{13}{9} \\ x &= \frac{\log_{10} \frac{13}{9}}{\log_{10} 2} \\ x &= 0.5305 \end{aligned}$$

Aufgabe 5.8

$$\begin{aligned} \log_{10} 5^{2x-3} &= \log_{10} 91 \\ (2x-3) \log_{10} 5 &= \log_{10} 91 \\ (2x-3) &= \frac{\log_{10} 91}{\log_{10} 5} \\ 2x &= \frac{\log_{10} 91}{\log_{10} 5} + 3 \\ x &= \frac{\log_{10} 91}{2 \log_{10} 5} + 1.5 \\ x &= 2.901 \end{aligned}$$

Aufgabe 5.9

$$\begin{aligned} \log_{10} 3^{\frac{1}{x}} &= \log_{10} 4 \\ \frac{1}{x} \cdot \log_{10} 3 &= \log_{10} 4 \\ \log_{10} 3 &= x \cdot \log_{10} 4 \\ x &= \frac{\log_{10} 3}{\log_{10} 4} \\ x &= 0.7925 \end{aligned}$$

Aufgabe 5.10

$$
\begin{aligned}
5 \cdot 3^x &= 912 \cdot 2^x && || : 2^x, : 5 \\
\frac{3^x}{2^x} &= \frac{912}{5} && \text{Potenzgesetze} \\
1.5^x &= 182.4 && || \log_{10} \\
\log_{10} 1.5^x &= \log_{10} 182.4 && \text{Logarithmengesetze} \\
x \cdot \log_{10} 1.5 &= \log_{10} 182.4 && || : \log_{10} 1.5 \\
x &= \frac{\log_{10} 182.4}{\log_{10} 1.5} \\
x &= 12.84
\end{aligned}
$$

$$
\begin{aligned}
5 \cdot 3^x &= 912 \cdot 2^x && || \log_{10} \\
\log_{10}(5 \cdot 3^x) &= \log_{10}(912 \cdot 2^x) && \text{Logarithmengesetze} \\
\log_{10} 5 + \log_{10} 3^x &= \log_{10} 912 + \log_{10} 2^x && \text{Logarithmengesetze} \\
\log_{10} 5 + x \log_{10} 3 &= \log_{10} 912 + x \log_{10} 2 && || - \log_{10} 5 - x \log_{10} 2 \\
x \log_{10} 3 - x \log_{10} 2 &= \log_{10} 912 - \log_{10} 5 && x \text{ ausklammern} \\
x(\log_{10} 3 - \log_{10} 2) &= \log_{10} 912 - \log_{10} 5 && || : (\dots) \\
x &= \frac{\log_{10} 912 - \log_{10} 5}{\log_{10} 3 - \log_{10} 2} \\
x &= 12.84
\end{aligned}
$$

Aufgabe 5.11

$$
\begin{aligned}
\log_{10} x^{\log_{10} x} &= \log_{10} 10^9 \\
\log_{10} x \cdot \log_{10} x &= 9 \\
(\log_{10} x)^2 &= 9 \\
\log_{10} x &= \pm 3 \\
x_1 &= 1000 \\
x_2 &= 0.001
\end{aligned}
$$

Aufgabe 5.12

$$
\begin{aligned}
5^x \cdot 5^1 + 5^x &= 12 \\
5^x(5+1) &= 12 \\
5^x &= 2 \\
\log_{10} 5^x &= \log_{10} 2 \\
x \cdot \log_{10} 5 &= \log_{10} 2 \\
x &= \frac{\log_{10} 2}{\log_{10} 5} \\
x &= 0.4307
\end{aligned}
$$

Aufgabe 5.13

$$
\begin{aligned}
4^x + 2^x &= 6 \\
(2^2)^x + 2^x &= 6 \\
(2^x)^2 + 2^x &= 6 \\
a^2 + a - 6 &= 0 \\
(a+3)(a-2) &= 0 \\
a_1 &= -3 = 2^x \text{ (keine Lösung)} \\
a_2 &= 2 = 2^x \\
x &= 1
\end{aligned}
$$

Aufgabe 5.14

$$
\begin{aligned}
9^x - 12 \cdot 3^x + 27 &= 0 \\
(3^x)^2 - 12 \cdot 3^x + 27 &= 0 \\
a^2 - 12 \cdot a + 27 &= 0 \\
(a-3)(a-9) &= 0 \\
a_1 &= 3 = 3^x \\
a_2 &= 9 = 3^x \\
x_1 &= 1 \\
x_2 &= 2
\end{aligned}
$$