

Aufgabe 1 (1P)

$$a_1 = -\frac{2}{3}, a_2 = -\frac{1}{5}, a_3 = 0, a_4 = \frac{1}{9}, a_5 = \frac{2}{11}$$

Aufgabe 2 (2P)

$$a_1 = 3 \quad (\text{gegeben})$$

$$a_2 = 1 \quad (\text{gegeben})$$

$$a_3 = a_2 + a_1 - 1 = 1 + 3 - 1 = 3$$

$$a_4 = a_3 + a_2 - 1 = 3 + 1 - 1 = 3$$

$$a_5 = a_4 + a_3 - 1 = 3 + 3 - 1 = 6$$

Aufgabe 3 (6P)

	beschränkt	monoton		alternierend
		wachsend	fallend	
$a_n = n^2$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
$b_n = \cos(n \cdot \pi)$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
$c_n = 1 - 0.5^n$	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
$d_n = (n+1)!/n!$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Aufgabe 4 (4P)

$$(a) a_n = \frac{n+2}{n+3} \xrightarrow{n \rightarrow \infty} 1 \quad \text{oder} \quad \lim_{n \rightarrow \infty} a_n = 1$$

$$(b) b_n = 5 + 2^{-n} \xrightarrow{n \rightarrow \infty} 5 \quad \text{oder} \quad \lim_{n \rightarrow \infty} b_n = 5$$

$$(c) c_n = \frac{n+1}{n \cdot (-1)^n} \text{ ist divergent}$$

$$(d) d_n = \frac{\sin(n \cdot \frac{\pi}{2})}{n} \xrightarrow{n \rightarrow \infty} 0 \quad \text{oder} \quad \lim_{n \rightarrow \infty} d_n = 0$$

Aufgabe 5 (4P)

$$(a) a_n = \frac{n+3n^2+2}{2n+2-4n^2} = \frac{(n^2+3n^2+2)/n^2}{(2n+2-4n^2)/n^2} = \frac{1/n+3+2/n^2}{2/n+2/n^2-4} \xrightarrow{n \rightarrow \infty} -\frac{3}{4}$$

$$(b) b_n = \frac{2n^4+7n^2}{3n^3+5n^5} = \frac{(2n^4+7n^2)/n^5}{(3n^3+5n^5)/n^5} = \frac{2/n+7/n^3}{3/n^2+5} \xrightarrow{n \rightarrow \infty} 0$$

$$\begin{aligned} \text{(c) } c_n &= \sqrt{2n+1} - \sqrt{2n} = \frac{(\sqrt{2n+1} - \sqrt{2n})(\sqrt{2n+1} + \sqrt{2n})}{\sqrt{2n+1} + \sqrt{2n}} \\ &= \frac{(2n+1) - 2n}{\sqrt{2n+1} + \sqrt{2n}} = \frac{1}{\sqrt{2n+1} + \sqrt{2n}} \xrightarrow{n \rightarrow \infty} 0 \end{aligned}$$