Aufgabe 1

(a) $y = b^x$

$8 = b^3$

$b = 2$

(b) $\frac{1}{81} = b^{-4}$

$81 = b^4$

$b = 3$

(c) $4 = b^{\frac{2}{3}}$

$4^3 = \left(b^{\frac{2}{3}}\right)^3$

$64 = b^2$

$b = 8$

(d) $\frac{2}{3} = b^{\frac{1}{2}}$

$\left(\frac{2}{3}\right)^2 = \left(b^{\frac{1}{2}}\right)^2$

$b = \frac{4}{9}$

(e) $\frac{4}{3} = b^{-\frac{1}{3}}$

$\left(\frac{4}{3}\right)^{-3} = \left(b^{-\frac{1}{3}}\right)^{-3}$

$\left(\frac{3}{4}\right)^3 = b$

$b = \frac{27}{64}$

Aufgabe 2

(a) $\frac{1}{8} = ab^{-1}$ (1) $8 = ab^2$ (2)

Dividiere (2) durch (1):

$8 : \frac{1}{8} = b^2 : b^{-1}$

$64 = b^3$

$b = 4$

Setze $b = 4$ z. B. in (1) ein:

$8 = a \cdot 16$

$a = \frac{1}{2}$

(b) $-4 = ab^{\frac{1}{2}} \quad (1) \qquad -2 = ab^{\frac{1}{4}} \quad (2)$

Dividiere (1) durch (2):

$\frac{-4}{-2} = \frac{ab^{\frac{1}{2}}}{ab^{\frac{1}{4}}}$

$2 = b^{\frac{1}{2}-\frac{1}{4}}$

$2 = b^{\frac{1}{4}}$

$b = 16$

Setze $b = 16$ z. B. in (1) ein:

$-4 = a \cdot 16^{\frac{1}{2}}$

$-4 = 4a$

$a = -1$

(c) $12 = ab^{-2} \quad (1) \qquad 24 = ab^{-3} \quad (2)$

Dividiere (1) durch (2):

$\frac{12}{24} = \frac{ab^{-2}}{ab^{-3}}$

$\frac{1}{2} = b^{-2-(-3)} = b^1$

$b = \frac{1}{2}$

Setze $b = \frac{1}{2}$ z. B. in (1) ein:

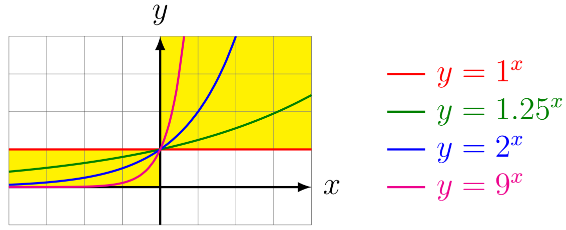
$12 = a\left(\frac{1}{2}\right)^{-2}$

$12 = a \cdot 2^2$

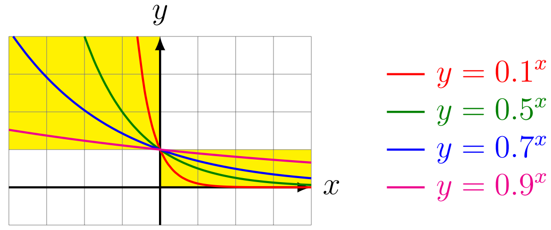
$a = 3$

Aufgabe 3

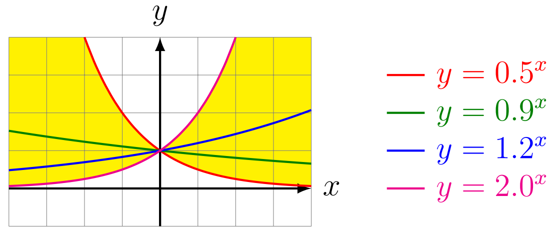
(a) $M = \{b \in \mathbb{R} \colon b \geq 1\}$



(b) $M = \{b \in \mathbb{R} \colon 0 < b \leq 1\}$

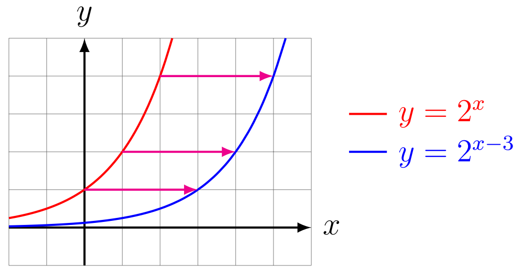


(c) $M = \{b \in \mathbb{R} \colon \frac{1}{2} < b \leq 2\}$

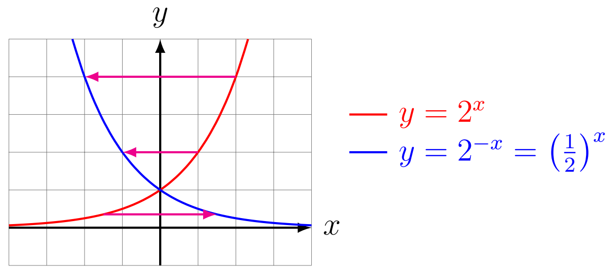


Aufgabe 4

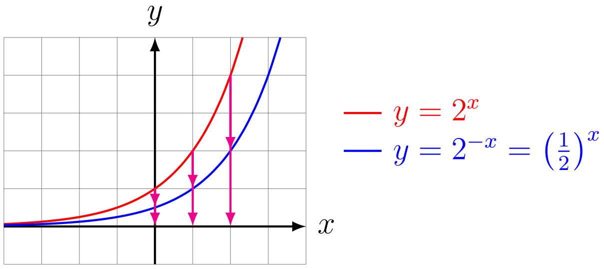
(a) Translation um 3 Einheiten in positive x-Richtung



(b) Achsenspiegelung an der y-Achse

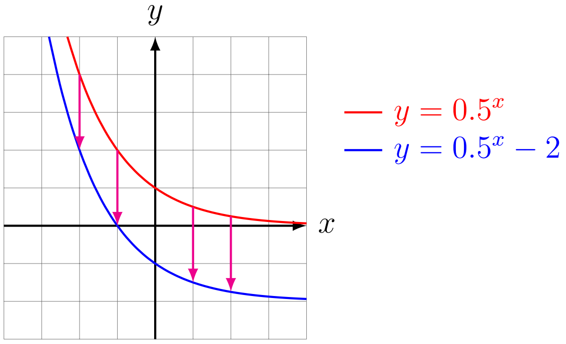


(c) Axiale Streckung senkrecht zur x-Achse mit dem Faktor $k = 0.5$

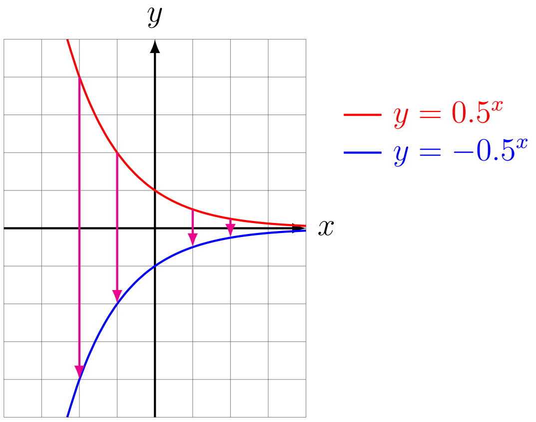


Aufgabe 5

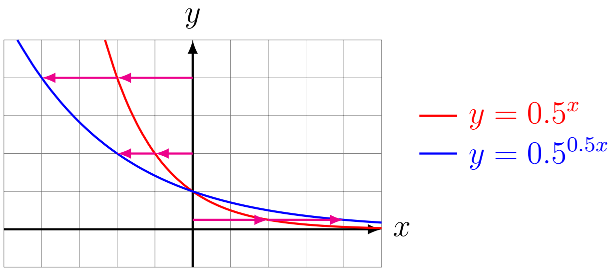
(a) Translation um 2 Einheiten in negative y-Richtung



(b) Achsenspiegelung an der x-Achse

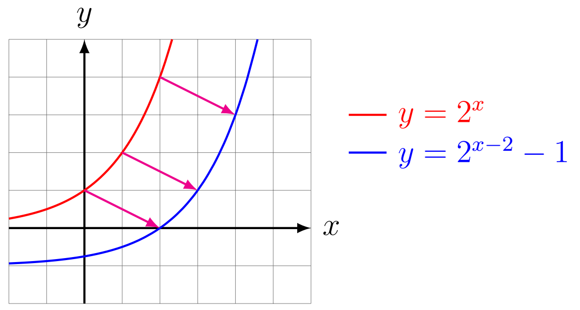


(c) Axiale Streckung senkrecht zur y-Achse mit dem Faktor $k = 2$

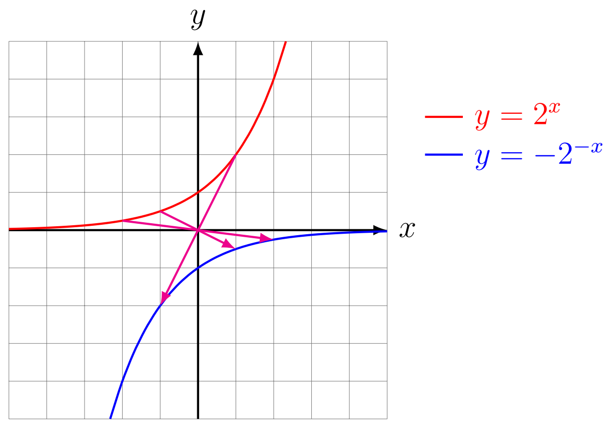


Aufgabe 6

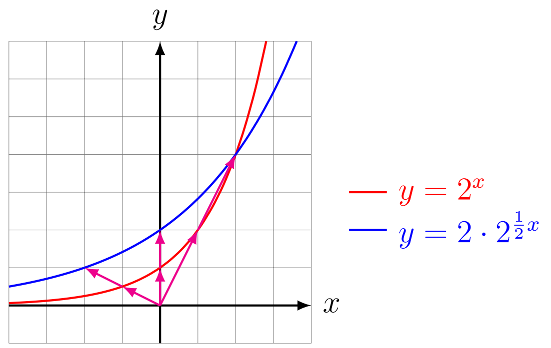
(a) Translation um den Vektor $\vec{v} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$: $x \to x - 2$, $y \to y - 1$



(b) Punktspiegelung am Ursprung: $x \to \frac{1}{2}x$, $y \to \frac{1}{2}y$



(c) Zentrische Streckung am Ursprung mit dem Faktor $k = 2$: $x \to \frac{1}{2}x$, $y \to \frac{1}{2}y$



Aufgabe 7*

(a) $x \to x - 2$, $y \to y - 1$

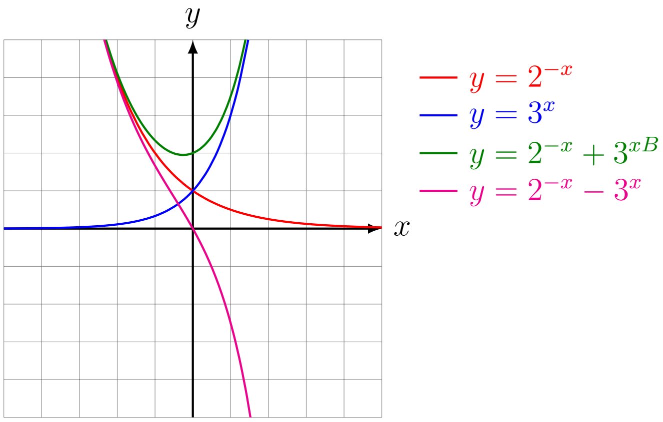
(b) Punktspiegelung am Ursprung: $x \to \frac{1}{2}x$, $y \to \frac{1}{2}y$

Aufgabe 8*

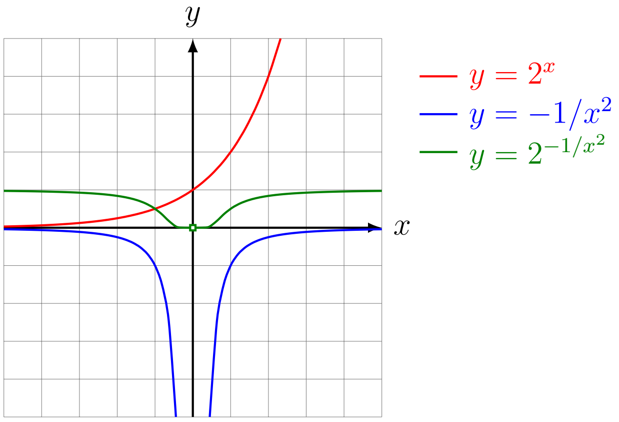
(a) $x \to x - 2$, $y \to y - 1$

(b) Punktspiegelung am Ursprung: $x \to \frac{1}{2}x$, $y \to \frac{1}{2}y$

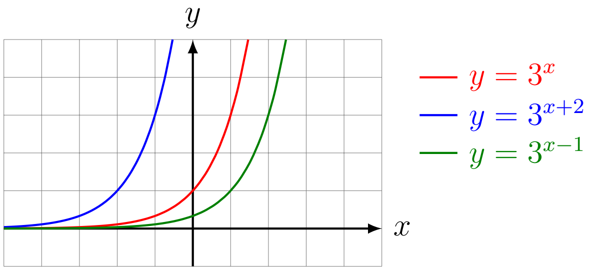
Aufgabe 9



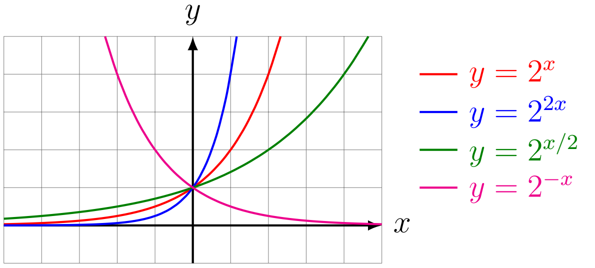
Aufgabe 10



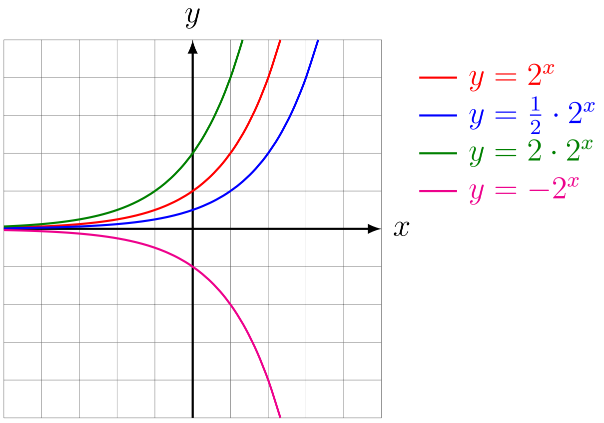
Aufgabe 11



Aufgabe 12



Aufgabe 13



Aufgabe 14

$K_8 = 40\,000 \cdot 1.035^8 = \text{CHF } 52\,672.35$

Aufgabe 15

(a) $K_{10} = K_0 \cdot 1.02^5 \cdot 1.03^5$

(b) $K_{10} = K_0 \cdot 1.03^5 \cdot 1.02^5$

Beide Varianten ergeben denselben Endwert (Kommutativgesetz).

Aufgabe 16

$K_0 = K_n : r^n$

$K_0 = 8\,000 : 1.02^5 = \text{CHF } 7245.85$

Aufgabe 17

$K_n = K_0 \cdot r^n \quad \Leftrightarrow \quad r = (K_n/K_0)^{\frac{1}{n}}$

$r = (5/4)^{\frac{1}{12}} = 1.018769 \quad \Rightarrow \quad p = 1.8769\%$

Aufgabe 18

$K_n = K_0 \cdot v^n$

$K_4 = K_0 \cdot 0.95^4 = 0.8145 \quad \Rightarrow \quad 81.45\%$

Aufgabe 19

$K_n = K_0 \cdot v^n$

$K_3 = K_0 \cdot 0.9^3 = 0.729 \quad \Rightarrow \quad 72.9\%$

Aufgabe 20

$K_n = K_0 \cdot v^n \quad \Leftrightarrow \quad v = (K_n/K_0)^{\frac{1}{n}}$

$v = (0.8/1)^{\frac{1}{6}} = 0.96349 = 1 - \frac{p}{100} \quad \Rightarrow \quad p \approx 3.4\%$

Aufgabe 21

Verwende eine Zeitskala die bei $t = 0$ (9:00 Uhr) beginnt.

$f(0) = a \cdot b^0 = a = 400$

$f(3) = a \cdot b^3 = 400 \cdot b^3 = 3200 \quad \Rightarrow \quad b^3 = 8 \quad \Rightarrow \quad b = 2$

(a) $P(2) = 400 \cdot 2^2 = 1600\, Bakterien$

(b) $P(3.5) = 400 \cdot 2^{3.5} = 4525\, Bakterien$

Aufgabe 22

Verwende eine Zeitskala die bei $t = 0$ (vor 4 Jahren) beginnt.

$f(0) = a \cdot b^0 = a = 11\,200$

$f(4) = a \cdot b^4 = 11\,200 \cdot b^4 = 56\,700 \quad \Rightarrow \quad b^4 = \dfrac{81}{16} \quad \Rightarrow \quad b = \frac{3}{2}$

(a) $P(4+5) = 11\,200 \cdot 1.5^9 = 430\,565\,\mathrm{m}^3$

(b) $P(4-6) = 11\,200 \cdot 1.5^{-2} = 4977\,\mathrm{m}^3$

Aufgabe 23

Aufgabe 24

Aufgabe 25

Aufgabe 26

Aufgabe 27

(a)

$$\begin{aligned} 3^{x+2} + 6 \cdot 3^{x+1} &= 1 \\ 3^{x+2} + 2 \cdot 3^1 \cdot 3^{x+1} &= 1 \\ 3^{x+2} + 2 \cdot 3^{x+2} &= 3^0 \\ 3 \cdot 3^{x+2} &= 3^0 \\ 3^{x+3} &= 3^0 \\ x + 3 &= 0 \\ x &= -3 \end{aligned}$$

(b)
$$\begin{aligned} 7 \cdot 2^{2x-4} - 4^{x-3} &= 1.5 \cdot 2^{3x+4} \\ 7 \cdot 2^{2x-4} - 2^{2x-6} &= 1.5 \cdot 2^{3x+4} \\ 7 \cdot 2^{2x-4} - 4 \cdot 2^{2x-4} &= 1.5 \cdot 2^{3x+4} \\ 3 \cdot 2^{2x-4} &= 3 \cdot 2^{3x+3} \\ 2x - 4 &= 3x + 3 \\ x &= -7 \end{aligned}$$

Aufgabe 28

(a)
$$\begin{aligned} 4^x - 4 &= 3 \cdot 2^x \\ 2^{2x} - 4 &= 3 \cdot 2^x \\ \left(2^x\right)^2 - 3 \cdot 2^x - 4 &= 0 \qquad \text{Substitution } 2^x = a \\ a^2 - 3a - 4 &= 0 \\ (a-4)(a+1) &= 0 \\ a_1 &= 4 = 2^2 = 2^{x_1} \\ a_2 &= -1 = 2^{x_2} \end{aligned}$$

$L = \{2\}$

(b)
$$\begin{aligned} 3 \cdot 9^x - 10 \cdot 3^x + 3 &= 0 \\ 3 \cdot 3^{2x} - 10 \cdot 3^x + 3 &= 0 \\ 3 \cdot \left(3^x\right)^2 - 10 \cdot 3^x + 3 &= 0 \qquad \text{Substitution } 3^x = a \\ 3a^2 - 10a + 3 &= 0 \end{aligned}$$

$D = (-10)^2 - 4 \cdot 3 \cdot 3 = 64$

$$a_1 = \frac{10 + \sqrt{64}}{6} = 3^1 = 3^{x_1}$$

$$a_2 = \frac{10 - \sqrt{64}}{6} = 1/3 = 3^{-1} = 3^{x_2}$$

$L = \{1, -1\}$

Aufgabe 29

(a)
$$\begin{aligned} 4^x - 4 &= 3 \cdot 2^x \\ 2^{2x} - 4 &= 3 \cdot 2^x \\ \left(2^x\right)^2 - 3 \cdot 2^x - 4 &= 0 \qquad \text{Substitution } 2^x = a \\ a^2 - 3a - 4 &= 0 \\ (a-4)(a+1) &= 0 \\ a_1 &= 4 = 2^2 = 2^{x_1} \\ a_2 &= -1 = 2^{x_2} \end{aligned}$$

$L = \{2\}$

(b)

$$3 \cdot 9^x - 10 \cdot 3^x + 3 = 0$$
$$3 \cdot 3^{2x} - 10 \cdot 3^x + 3 = 0$$
$$3 \cdot \left(3^x\right)^2 - 10 \cdot 3^x + 3 = 0 \qquad \text{Substitution } 3^x = a$$
$$3a^2 - 10a + 3 = 0$$

$$D = (-10)^2 - 4 \cdot 3 \cdot 3 = 64$$

$$a_1 = \frac{10 + \sqrt{64}}{6} = 3^1 = 3^{x_1}$$

$$a_2 = \frac{10 - \sqrt{64}}{6} = 1/3 = 3^{-1} = 3^{x_2}$$

$$L = \{1, -1\}$$