
Quadratwurzeln
Lösungen+ (L+)

Aufgabe 1.1

$$(a) \sqrt{144} = 12$$

$$(e) \sqrt{225} = 15$$

$$(b) \sqrt{324} = 18$$

$$(f) \sqrt{400} = 20$$

$$(c) \sqrt{529} = 23$$

$$(g) \sqrt{256} = 16$$

$$(d) \sqrt{576} = 24$$

$$(h) \sqrt{625} = 25$$

Aufgabe 1.2

$$(a) \sqrt{119250736929}: 6 \text{ Vor- und } 0 \text{ Nachkommastellen}$$

$$(b) \sqrt{36692168704}: 6 \text{ Vor- und } 0 \text{ Nachkommastellen}$$

$$(c) \sqrt{279016.26275601}: 3 \text{ Vor- und } 4 \text{ Nachkommastellen}$$

$$(d) \sqrt{0.0000184041}: 0 \text{ Vor- und } 5 \text{ Nachkommastellen}$$

Aufgabe 1.3

$$\begin{array}{r} 22 \text{ 09} = (40 + 7)^2 \\ 16 \qquad 4^2 \\ \hline 6 \text{ 09} \\ 6 \text{ 09} \quad (2 \cdot 40 + 7) \cdot 7 \\ \hline 0 \end{array}$$

$$\sqrt{2209} = 47$$

Aufgabe 1.4

$$\begin{array}{r} 30 \text{ 25} = (50 + 5)^2 \\ 25 \qquad 5^2 \\ \hline 5 \text{ 25} \\ 5 \text{ 25} \quad (2 \cdot 50 + 5) \cdot 5 \\ \hline 0 \end{array}$$

$$\sqrt{3025} = 55$$

Aufgabe 1.5

$$\begin{array}{r} 5 \text{ 33 61} = (200 + 30 + 1)^2 \\ 4 \qquad 2^2 \\ \hline 1 \text{ 33} \\ 1 \text{ 29} \quad (2 \cdot 20 + 3) \cdot 3 \\ \hline 0 \text{ 04 61} \\ 04 \text{ 61} \quad (2 \cdot 200 + 2 \cdot 30 + 1) \cdot 1 \\ \hline 0 \end{array}$$

$$\sqrt{53361} = 231$$

Aufgabe 1.6

$$\begin{array}{r} 32\ 03\ 56 = (500 + 60 + 6)^2 \\ \underline{25} \qquad 5^2 \\ 7\ 03 \\ \underline{6\ 36} \qquad (2 \cdot 50 + 6) \cdot 6 \\ 0\ 67\ 56 \\ \underline{67\ 56} \qquad (2 \cdot 500 + 2 \cdot 60 + 6) \cdot 6 \\ 0 \end{array}$$
$$\sqrt{320356} = 566$$

Aufgabe 1.7

$$x_2 = \frac{1}{2} \left(x_1 + \frac{a}{x_1} \right) = \frac{1}{2} \left(\frac{3}{4} + \frac{5}{6} \right) = \frac{19}{24}$$
$$|x_2 - x_1| = \left| \frac{19}{24} - \frac{3}{4} \right| = \frac{1}{24} < \frac{1}{10}$$

Näherung: $\sqrt{\frac{5}{8}} \approx \frac{19}{24}$

Aufgabe 1.8

$$x_2 = \frac{1}{2} \left(x_1 + \frac{a}{x_1} \right) = \frac{1}{2} \left(2 + \frac{3}{2} \right) = \frac{7}{4}$$
$$|x_2 - x_1| = \left| \frac{7}{4} - 2 \right| = \frac{1}{4} \geq \frac{1}{10}$$

$x_2 \rightarrow x_1$

$$x_2 = \frac{1}{2} \left(x_1 + \frac{a}{x_1} \right) = \frac{1}{2} \left(\frac{7}{4} + \frac{12}{7} \right) = \frac{97}{56}$$
$$|x_2 - x_1| = \left| \frac{97}{56} - \frac{7}{4} \right| = \frac{1}{56} < \frac{1}{10}$$

Näherung: $\sqrt{3} \approx \frac{97}{56}$

Aufgabe 1.9

$$x_2 = \frac{1}{2} \left(x_1 + \frac{a}{x_1} \right) = \frac{1}{2} \left(1 + \frac{1}{2} \right) = \frac{3}{4}$$
$$|x_2 - x_1| = \left| \frac{3}{4} - 1 \right| = \frac{1}{4} \geq \frac{1}{10}$$

$x_2 \rightarrow x_1$

$$x_2 = \frac{1}{2} \left(x_1 + \frac{a}{x_1} \right) = \frac{1}{2} \left(\frac{3}{4} + \frac{2}{3} \right) = \frac{17}{24}$$
$$|x_2 - x_1| = \left| \frac{17}{24} - \frac{3}{4} \right| = \frac{1}{24} < \frac{1}{10}$$

Näherung: $\sqrt{\frac{1}{2}} \approx \frac{17}{24}$

Aufgabe 2.1

(a) $\sqrt{6\,400\,000\,000} = 80\,000$

(c) $\sqrt{19\,600\,000\,000} = 140\,000$

(b) $\sqrt{36\,100\,000\,000} = 190\,000$

(d) $\sqrt{5\,760\,000} = 2\,400$

Aufgabe 2.2

(a) $\sqrt{0.0049} = 0.07$

(b) $\sqrt{0.0000000256} = 0.00016$

(c) $\sqrt{0.00000324} = 0.0018$

(d) $\sqrt{0.000529} = 0.023$

Aufgabe 2.3

(a) $\sqrt{4 \cdot 10^{26}} = 2 \cdot 10^{13}$

(c) $\sqrt{6.4 \cdot 10^{53}} = 8 \cdot 10^{26}$

(b) $\sqrt{2.25 \cdot 10^{26}} = 1.5 \cdot 10^{13}$

(d) $\sqrt{5.76 \cdot 10^{68}} = 2.4 \cdot 10^{34}$

Aufgabe 2.4

(a) $\sqrt{9 + 16} = \sqrt{9} + \sqrt{16}$

$$\sqrt{25} = 3 + 4$$

$$5 = 7 \quad \text{falsch}$$

(b) $\sqrt{25 + 144} = \sqrt{25} + \sqrt{144}$

$$\sqrt{169} = 5 + 12$$

$$13 = 17 \quad \text{falsch}$$

(c) $\sqrt{49 + 576} = \sqrt{49} + \sqrt{576}$

$$\sqrt{625} = 7 + 24$$

$$25 = 31 \quad \text{falsch}$$

Radizieren und Addieren dürfen nicht vertauscht werden.

Aufgabe 2.5

(a) $\sqrt{5\frac{19}{25}} = \sqrt{\frac{144}{25}} = \frac{12}{5}$

(b) $\sqrt{30\frac{1}{4}} = \sqrt{\frac{121}{4}} = \frac{11}{2}$

(c) $\sqrt{2\frac{14}{25}} = \sqrt{\frac{64}{25}} = \frac{8}{5}$

Aufgabe 2.6

- (a) $\sqrt{64 \cdot 121} = \sqrt{64} \cdot \sqrt{121} = 8 \cdot 11 = 88$
(b) $\sqrt{81 \cdot 484} = \sqrt{81} \cdot \sqrt{484} = 9 \cdot 22 = 198$
(c) $\sqrt{36 \cdot 324} = \sqrt{36} \cdot \sqrt{324} = 6 \cdot 18 = 108$

Aufgabe 2.7

- (a) $\sqrt{2} \cdot \sqrt{8} = \sqrt{2 \cdot 8} = \sqrt{2 \cdot 2 \cdot 2} = 4$
(b) $\sqrt{2} \cdot \sqrt{18} = \sqrt{2 \cdot 18} = \sqrt{2 \cdot 2 \cdot 3} = 6$
(c) $\sqrt{5} \cdot \sqrt{45} = \sqrt{5 \cdot 45} = \sqrt{5 \cdot 5 \cdot 3} = 15$

Aufgabe 2.8

- (a) $\sqrt{\frac{100}{529}} = \frac{\sqrt{100}}{\sqrt{529}} = \frac{10}{23}$
(b) $\sqrt{\frac{100}{169}} = \frac{\sqrt{100}}{\sqrt{169}} = \frac{10}{13}$
(c) $\sqrt{\frac{100}{289}} = \frac{\sqrt{100}}{\sqrt{289}} = \frac{10}{17}$

Aufgabe 2.9

- (a) $\frac{\sqrt{150}}{\sqrt{54}} = \sqrt{\frac{150}{54}} = \sqrt{\frac{6 \cdot 25}{6 \cdot 9}} = \sqrt{\frac{25}{9}} = \frac{\sqrt{25}}{\sqrt{9}} = \frac{5}{3}$
(b) $\frac{\sqrt{567}}{\sqrt{343}} = \sqrt{\frac{567}{343}} = \sqrt{\frac{7 \cdot 81}{7 \cdot 49}} = \sqrt{\frac{81}{49}} = \frac{\sqrt{81}}{\sqrt{49}} = \frac{9}{7}$
(c) $\frac{\sqrt{75}}{\sqrt{108}} = \sqrt{\frac{75}{108}} = \sqrt{\frac{3 \cdot 25}{3 \cdot 36}} = \sqrt{\frac{25}{36}} = \frac{\sqrt{25}}{\sqrt{36}} = \frac{5}{6}$

Aufgabe 2.10

- (a) $\sqrt{(-116)^2} = 116$
(b) $\sqrt{\frac{19}{60}} \cdot \sqrt{\frac{19}{60}} = \frac{19}{60}$
(c) $\sqrt{(1 - 10^{11})^2} = \sqrt{-(10^{11} - 1)^2} = 10^{11} - 1$

Aufgabe 2.11

$$(a) \sqrt{\sqrt{81}} = \sqrt{9} = \sqrt{3}$$

$$(b) \sqrt{64 + \sqrt{289}} = \sqrt{64 + 17} = \sqrt{81} = \sqrt{9}$$

$$(c) \sqrt{72 + \sqrt{68 + \sqrt{169}}} = \sqrt{72 + \sqrt{68 + 13}} = \sqrt{72 + \sqrt{81}} \\ = \sqrt{72 + 9} = \sqrt{81} = 9$$

Aufgabe 2.12

$$(a) \sqrt{2^{93} \cdot 2^{93}} = 2^{93}$$

$$(b) \sqrt{2^{93} : 2^{93}} = \sqrt{1} = 1$$

$$(c) \sqrt{2^{93} - 2^{93}} = \sqrt{0} = 0$$

$$(d) \sqrt{2^{93} + 2^{93}} = \sqrt{2 \cdot 2^{93}} = \sqrt{2^{94}} = 2^{47}$$

Aufgabe 2.13

$$(a) \sqrt{0.0045} \cdot \sqrt{450000} = \sqrt{45} \cdot \sqrt{45} = 45$$

$$(b) \sqrt{0.0003} \cdot \sqrt{30000} = \sqrt{3} \cdot \sqrt{3} = 3$$

$$(c) \sqrt{0.092} \cdot \sqrt{920} = \sqrt{9.2} \cdot \sqrt{9.2} = 9.2$$

$$(d) \sqrt{11.5} \cdot \sqrt{1150} = \sqrt{115} \cdot \sqrt{115} = 115$$

Aufgabe 2.14

$$(a) \sqrt{10} \cdot \sqrt{4.9} = \sqrt{10 \cdot 4.9} = \sqrt{49} = 7$$

$$(b) \frac{\sqrt{361000}}{\sqrt{1000}} = \sqrt{\frac{361000}{1000}} = \sqrt{361} = 19$$

$$(c) \sqrt{1000} \cdot \sqrt{0.121} = \sqrt{1000 \cdot 0.121} = \sqrt{121} = 11$$

$$(d) \frac{\sqrt{5290}}{\sqrt{10}} = \sqrt{\frac{5290}{10}} = \sqrt{529} = 23$$

Aufgabe 2.15

$$(a) \sqrt{7} \cdot \sqrt{7} \cdot \sqrt{7} \cdot \sqrt{7} = 49$$

$$(b) \sqrt{6} \cdot \sqrt{6} \cdot \sqrt{6} \cdot \sqrt{6} = 36$$

Aufgabe 2.16

$$(a) \sqrt{3} \cdot \sqrt{15} \cdot \sqrt{8} \cdot \sqrt{10} = \sqrt{3} \cdot \sqrt{3} \cdot \sqrt{5} \cdot \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{5} = 3600$$

$$(b) \sqrt{6} \cdot \sqrt{5} \cdot \sqrt{8} \cdot \sqrt{15} = \sqrt{2} \cdot \sqrt{3} \cdot \sqrt{5} \cdot \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{3} \cdot \sqrt{5} = 3600$$

Aufgabe 2.17

$$(a) \sqrt{0.5} \cdot \sqrt{50} = \sqrt{0.5} \cdot \sqrt{10} \cdot \sqrt{5} = \sqrt{5} \cdot \sqrt{5} = 5$$

$$(b) \sqrt{38\,000} \cdot \sqrt{3.8} = \sqrt{380} \cdot \sqrt{100} \cdot \sqrt{3.8} = \sqrt{380} \cdot \sqrt{380} = 380$$

$$(c) \sqrt{0.123} \cdot \sqrt{0.00123} = \sqrt{0.0123} \cdot \sqrt{10} \cdot \sqrt{0.00123} \\ = \sqrt{0.0123} \cdot \sqrt{0.0123} = 0.0123$$

Aufgabe 2.18

$$(a) \sqrt{\frac{7}{65}} \sqrt{\frac{13}{35}} = \sqrt{\frac{7}{65} \cdot \frac{13}{35}} = \sqrt{\frac{7 \cdot 13}{5 \cdot 13 \cdot 5 \cdot 7}} = \sqrt{\frac{1}{5 \cdot 5}} = \frac{1}{5}$$

$$(b) \sqrt{\frac{1}{2}} \sqrt{\frac{2}{3}} \sqrt{\frac{3}{4}} = \sqrt{\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

Aufgabe 2.19

$$(a) (3\sqrt{2})^2 = 9 \cdot 2 = 18$$

$$(b) (2\sqrt{5})^2 = 4 \cdot 5 = 20$$

$$(c) (7\sqrt{3})^2 = 49 \cdot 3 = 147$$

Aufgabe 2.20

$$(a) \sqrt{3} \cdot \frac{8}{3} \cdot \sqrt{3} = \sqrt{3} \cdot \sqrt{3} \cdot \frac{8}{3} = 3 \cdot \frac{8}{3} = 8$$

$$(b) \frac{\sqrt{12}}{3} \cdot \frac{\sqrt{12}}{4} = \frac{\sqrt{12} \cdot \sqrt{12}}{3 \cdot 4} = \frac{12}{12} = 1$$

$$(c) \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{\sqrt{2}^2}{2^2} = \frac{2}{4} = \frac{1}{2}$$

Aufgabe 2.21

$$(a) \sqrt{\sqrt{7}} \cdot \sqrt{7\sqrt{7}} = \sqrt{\sqrt{7} \cdot 7 \cdot \sqrt{7}} = \sqrt{7 \cdot 7} = 7$$

$$(b) \sqrt{5\sqrt{3}} \cdot \sqrt{15\sqrt{3}} = \sqrt{5\sqrt{3} \cdot 15 \cdot \sqrt{3}} = \sqrt{5 \cdot 15 \cdot 3} \\ = \sqrt{15 \cdot 15} = 15$$

$$(c) \sqrt{\sqrt{3}\sqrt{12}}\sqrt{\sqrt{2}\sqrt{18}} = \sqrt{\sqrt{36} \cdot \sqrt{36}} = \sqrt{36} = 6$$

Achtung: falsche Aufgabenstellung in (c)

Aufgabe 2.22

$$(a) \sqrt{3}(\sqrt{3} + 1) = \sqrt{3} \cdot \sqrt{3} + \sqrt{3} \cdot 1 = 3 + \sqrt{3}$$

$$(b) \sqrt{2}(\sqrt{8} - \sqrt{2}) = \sqrt{2} \cdot \sqrt{8} - \sqrt{2} \cdot \sqrt{2} = 4 - 2 = 2$$

$$(c) \sqrt{10} \left(\sqrt{\frac{1}{5}} + \sqrt{\frac{1}{2}} \right) = \sqrt{10} \cdot \sqrt{\frac{1}{5}} + \sqrt{10} \cdot \sqrt{\frac{1}{2}} = \sqrt{2} + \sqrt{5}$$

Aufgabe 2.23

$$(a) (3 + \sqrt{2})(3 - \sqrt{2}) = 9 - 2 = 7$$

$$(b) (4 - \sqrt{3})(4 + \sqrt{3}) = 16 - 3 = 13$$

$$(c) (\sqrt{7} - \sqrt{5})(\sqrt{7} + \sqrt{5}) = 7 - 5 = 2$$

Aufgabe 2.24

$$(a) \sqrt{3 + 2\sqrt{2}} \sqrt{3 - 2\sqrt{2}} = \sqrt{3^2 - 2^2 \cdot 2} = \sqrt{1} = 1$$

$$(b) \sqrt{7 + 2\sqrt{6}} \sqrt{7 - 2\sqrt{6}} = \sqrt{7^2 - 2^2 \cdot 6} = \sqrt{25} = 5$$

Aufgabe 2.25

$$(a) \left(\sqrt{7 + 3\sqrt{5}} + \sqrt{7 - 3\sqrt{5}} \right)^2 \\ = (7 + 3\sqrt{5}) + 2\sqrt{7 + 3\sqrt{5}}\sqrt{7 - 3\sqrt{5}} + (7 - 3\sqrt{5}) \\ = 14 + 2\sqrt{(7 + 3\sqrt{5})(7 - 3\sqrt{5})} \\ = 14 + 2\sqrt{49 - 45} = 14 + 2\sqrt{4} = 18$$

$$\begin{aligned}
\text{(b)} \quad & \left(\sqrt{8 + 2\sqrt{7}} - \sqrt{8 - 2\sqrt{7}} \right)^2 \\
&= (8 + 2\sqrt{7}) - 2\sqrt{8 + 2\sqrt{7}}\sqrt{8 - 2\sqrt{7}} + (8 - 2\sqrt{7}) \\
&= 16 - 2\sqrt{(8 + 2\sqrt{7})(8 - 2\sqrt{7})} \\
&= 16 - 2\sqrt{64 - 28} = 16 - 2\sqrt{36} = 16 - 12 = 4
\end{aligned}$$

Aufgabe 2.26

$$\begin{aligned}
\text{(a)} \quad & 2\sqrt{7} = \sqrt{4}\sqrt{7} = \sqrt{28} \\
\text{(b)} \quad & 2\sqrt{\frac{1}{2}} = \sqrt{4}\sqrt{\frac{1}{2}} = \sqrt{2} \\
\text{(c)} \quad & \frac{1}{5}\sqrt{5} = \sqrt{125}\sqrt{5} = \sqrt{\frac{1}{5}} \\
\text{(d)} \quad & 10\sqrt{0.31} = \sqrt{100} \cdot \sqrt{0.31} = \sqrt{31} \\
\text{(e)} \quad & \frac{2}{3}\sqrt{\frac{3}{2}} = \sqrt{\frac{4}{9}}\sqrt{\frac{3}{2}} = \sqrt{\frac{2}{3}}
\end{aligned}$$

Aufgabe 3.1

$$\begin{aligned}
\text{(a)} \quad & \sqrt{28} = 2\sqrt{7} \\
\text{(b)} \quad & \sqrt{48} = 4\sqrt{3} \\
\text{(c)} \quad & \sqrt{92} = 2\sqrt{23} \\
\text{(d)} \quad & \sqrt{98} = 7\sqrt{2}
\end{aligned}$$

Aufgabe 3.2

$$\begin{aligned}
\text{(a)} \quad & \sqrt{162} = 9\sqrt{2} \\
\text{(b)} \quad & \sqrt{604} = 2\sqrt{151} \\
\text{(c)} \quad & \sqrt{882} = 21\sqrt{2} \\
\text{(d)} \quad & \sqrt{925} = 5\sqrt{37}
\end{aligned}$$

Aufgabe 3.3

$$\begin{aligned}
\text{(a)} \quad & \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{3}}{3} = \frac{1}{3}\sqrt{3} \\
\text{(b)} \quad & \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{\sqrt{5}}{5} = \frac{1}{5}\sqrt{5} \\
\text{(c)} \quad & \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{2\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2}}{4} = \frac{1}{4}\sqrt{2}
\end{aligned}$$

Aufgabe 3.4

$$(a) \sqrt{\frac{1}{7}} = \frac{1}{\sqrt{7}} = \frac{\sqrt{7}}{\sqrt{7} \cdot \sqrt{7}} = \frac{\sqrt{7}}{7} = \frac{1}{7}\sqrt{7}$$

$$(b) \sqrt{\frac{2}{11}} = \frac{\sqrt{2}}{\sqrt{11}} = \frac{\sqrt{2} \cdot \sqrt{11}}{\sqrt{11} \cdot \sqrt{11}} = \frac{\sqrt{22}}{11} = \frac{1}{11}\sqrt{22}$$

$$(c) \sqrt{\frac{1}{18}} = \frac{1}{3\sqrt{2}} = \frac{\sqrt{2}}{3\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2}}{6} = \frac{1}{6}\sqrt{2}$$

Aufgabe 3.5

$$(a) \frac{\sqrt{12}}{\sqrt{20}} = \frac{2\sqrt{3}}{2\sqrt{5}} = \frac{\sqrt{3} \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{\sqrt{15}}{5} = \frac{1}{5}\sqrt{15}$$

$$(b) \frac{\sqrt{16}}{\sqrt{50}} = \frac{4}{5\sqrt{2}} = \frac{4\sqrt{2}}{5\sqrt{2} \cdot \sqrt{2}} = \frac{4\sqrt{2}}{10} = \frac{2}{5}\sqrt{2}$$

$$(c) \frac{\sqrt{33}}{\sqrt{44}} = \frac{\sqrt{11} \cdot \sqrt{3}}{2\sqrt{11}} = \frac{\sqrt{3}}{2} = \frac{1}{2}\sqrt{3}$$

Aufgabe 3.6

$$(a) \sqrt{50} + \sqrt{75} + \sqrt{32} = 5\sqrt{2} + 5\sqrt{3} + 4\sqrt{2} = 9\sqrt{2} + 5\sqrt{3}$$

$$(b) \sqrt{45} - \sqrt{180} + \sqrt{125} + \sqrt{16} = 3\sqrt{5} - 5\sqrt{6} + 5\sqrt{5} + 4 \\ = 4 + 8\sqrt{5} - 5\sqrt{6}$$

$$(c) \sqrt{72} + \sqrt{80} + \sqrt{245} - \sqrt{18} = 6\sqrt{2} + 4\sqrt{5} + 7\sqrt{5} - 3\sqrt{2} \\ = 3\sqrt{2} + 11\sqrt{5}$$

Aufgabe 3.7

$$(a) \frac{4}{\sqrt{28}} + \frac{3}{\sqrt{150}} - \frac{5}{\sqrt{7}} = \frac{4}{2\sqrt{7}} + \frac{3}{5\sqrt{6}} - \frac{5}{\sqrt{7}} \\ = \frac{2}{\sqrt{7}} + \frac{3}{5\sqrt{6}} - \frac{5}{\sqrt{7}} = \frac{3}{5\sqrt{6}} - \frac{3}{\sqrt{7}} \\ = \frac{3\sqrt{6}}{5\sqrt{6}\sqrt{6}} - \frac{3\sqrt{7}}{\sqrt{7}\sqrt{7}} = \frac{3\sqrt{6}}{30} - \frac{3\sqrt{7}}{7} = \frac{1}{10}\sqrt{6} - \frac{3}{7}\sqrt{7}$$

$$\begin{aligned}
\text{(b)} \quad & \frac{1}{\sqrt{27}} - \frac{6}{\sqrt{24}} + \frac{3}{\sqrt{54}} + \frac{10}{\sqrt{25}} = \frac{1}{3\sqrt{3}} - \frac{6}{2\sqrt{6}} + \frac{3}{3\sqrt{6}} + 2 \\
& = 2 + \frac{1}{3\sqrt{3}} - \frac{3}{\sqrt{6}} + \frac{1}{\sqrt{6}} = 2 + \frac{1}{3\sqrt{3}} - \frac{2}{\sqrt{6}} \\
& = 2 + \frac{\sqrt{3}}{9} - \frac{2\sqrt{6}}{6} = 2 + \frac{1}{9}\sqrt{3} - \frac{1}{3}\sqrt{6}
\end{aligned}$$

Aufgabe 3.8

$$\text{(a)} \quad (\sqrt{3} + \sqrt{6})^2 = 3 + 2\sqrt{3}\sqrt{6} + 6 = 9 + 2\sqrt{18} = 9 + 6\sqrt{3}$$

$$\text{(b)} \quad (2\sqrt{3} - \sqrt{2})^2 = 12 - 4\sqrt{6} + 2 = 14 - 4\sqrt{6}$$

Aufgabe 3.9

$$\text{(a)} \quad (3 + \sqrt{50})^2 = 9 + 2 \cdot 3 \cdot 5\sqrt{2} + 50 = 59 + 30\sqrt{2}$$

$$\text{(b)} \quad (\sqrt{6} + \sqrt{24})^2 = (\sqrt{6} + 2\sqrt{6})^2 = (3\sqrt{6})^2 = 9 \cdot 6 = 54$$

Aufgabe 3.10

$$\begin{aligned}
\text{(a)} \quad & (1 + \sqrt{2} + \sqrt{5})^2 = 1 + 2 + 5 + 2\sqrt{2} + 2\sqrt{5} + 2\sqrt{10} \\
& = 8 + 2\sqrt{2} + 2\sqrt{5} + 2\sqrt{10}
\end{aligned}$$

$$\begin{aligned}
\text{(b)} \quad & (1 + \sqrt{2} - \sqrt{5})(1 + \sqrt{2} + \sqrt{5}) \stackrel{\text{3BF}}{=} (1 + \sqrt{2})^2 - 5 \\
& = 1 + 2\sqrt{2} + 2 - 5 \\
& = -2 + 2\sqrt{2}
\end{aligned}$$

Aufgabe 3.11

$$\text{(a)} \quad \frac{3 + \sqrt{3}}{\sqrt{3}} = \frac{(3 + \sqrt{3}) \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{3\sqrt{3} + 3}{3} = 1 + \sqrt{3}$$

$$\begin{aligned}
\text{(b)} \quad & \frac{\sqrt{2} + \sqrt{10}}{\sqrt{5}} = \frac{(\sqrt{2} + \sqrt{10}) \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{\sqrt{10} + \sqrt{50}}{5} \\
& = \frac{\sqrt{10} + 5\sqrt{2}}{5} = \sqrt{2} + \frac{1}{5}\sqrt{10}
\end{aligned}$$

Aufgabe 3.12

$$(a) \frac{1}{2 + \sqrt{3}} = \frac{(2 - \sqrt{3})}{(2 + \sqrt{3})(2 - \sqrt{3})} = \frac{(2 - \sqrt{3})}{4 - 3} = 2 - \sqrt{3}$$

$$(b) \frac{6}{1 - 2\sqrt{2}} = \frac{6(1 + 2\sqrt{2})}{(1 - 2\sqrt{2})(1 + 2\sqrt{2})} = \frac{6 + 12\sqrt{2}}{1 - 8} \\ = -\frac{6}{7} - \frac{12}{7}\sqrt{2}$$

Aufgabe 3.13

$$(a) \frac{8 - \sqrt{7}}{3 - \sqrt{5}} = \frac{(8 - \sqrt{7})(3 + \sqrt{5})}{(3 - \sqrt{5})(3 + \sqrt{5})} = \frac{24 + 8\sqrt{5} - 3\sqrt{7} - \sqrt{35}}{4} \\ = 6 + 2\sqrt{5} - \frac{3}{4}\sqrt{7} - \frac{1}{4}\sqrt{35}$$

$$(b) \frac{\sqrt{5} - 2\sqrt{2}}{\sqrt{5} + 2\sqrt{2}} = \frac{(\sqrt{5} - 2\sqrt{2})(\sqrt{5} - 2\sqrt{2})}{(\sqrt{5} + 2\sqrt{2})(\sqrt{5} - 2\sqrt{2})} = \frac{5 - 4\sqrt{10} + 8}{5 - 8} \\ = \frac{13 - 4\sqrt{10}}{-3} = -\frac{13}{3} + \frac{4}{3}\sqrt{10}$$

Aufgabe 3.14

$$\frac{5}{\sqrt{5}} - \frac{2}{1 - \sqrt{2}} + \frac{4 + \sqrt{3}}{\sqrt{3}} \\ = \frac{5 \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} - \frac{2 \cdot (1 + \sqrt{2})}{(1 - \sqrt{2}) \cdot (1 + \sqrt{2})} + \frac{(4 + \sqrt{3}) \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} \\ = \frac{5\sqrt{5}}{5} - \frac{2 + 2\sqrt{2}}{1 - 2} + \frac{4\sqrt{3} + 3}{3} \\ = \sqrt{5} + 2 + 2\sqrt{2} + \frac{4}{3}\sqrt{3} + 1 \\ = 3 + 2\sqrt{2} + \frac{4}{3}\sqrt{3} + \sqrt{5}$$

Aufgabe 4.1

$$(a) \sqrt{p^2} = |p|$$

$$(b) \sqrt{9u^4} = 3u^2 \text{ (kein Betrag nötig)}$$

$$(c) \sqrt{k^7} = \sqrt{k^6 \cdot k} = \sqrt{k^6} \sqrt{k} = |k^3| \sqrt{k}$$

$$(d) \sqrt{32z^{13}} = \sqrt{16z^{12} \cdot 2z} = \sqrt{16z^{12}} \sqrt{2z} = 4z^6 \sqrt{2z}$$

Aufgabe 4.2

- (a) $\sqrt{a^2b^6} = |ab^3|$
- (b) $\sqrt{5s^3t^4} = \sqrt{5s \cdot s^2 \cdot t^4} = |s|t^2\sqrt{5s}$
- (c) $\sqrt{e^{16}m^{20}} = e^8m^{10}$
- (d) $\sqrt{x^7y^9} = \sqrt{x^6y^8 \cdot xy} = |x|^3y^4\sqrt{xy}$

Aufgabe 4.3

- (a) $\sqrt{\frac{a^2}{b^2}} = \frac{|a|}{|b|}$
- (b) $\sqrt{\frac{100m^4}{n^2}} = \frac{10m^2}{|n|}$
- (c) $\sqrt{\frac{p^4}{q^8}} = \frac{p^2}{q^4}$
- (d) $\sqrt{\frac{18v^{11}}{v^3}} = \sqrt{18v^8} = 3v^4\sqrt{2}$

Aufgabe 4.4

- (a) $\sqrt{x^2 + 4x + 4} = \sqrt{(x+2)^2} = |x+2|$
- (b) $\sqrt{a^2 - 22a + 121} = \sqrt{(a-11)^2} = |a-11|$
- (c) $\sqrt{20m^2 + 40m + 20} = \sqrt{20(m^2 + 2m + 1)}$
 $= \sqrt{4 \cdot 5(m+1)^2} = 2|m+1|\sqrt{5}$

Aufgabe 4.5

- (a) $\sqrt{(xy)^2 + (xy)^2 + (xy)^2} = \sqrt{3(xy)^2} = |xy|\sqrt{3}$
- (b) $\sqrt{k^7 + k^6} = \sqrt{k^6(k+1)} = |k^3|\sqrt{k+1}$
- (c) $\sqrt{(r+s)^2 - 4rs} = \sqrt{r^2 + 2rs + s^2 - 4rs} = \sqrt{r^2 - 2rs + s^2}$
 $= \sqrt{(r-s)^2} = |r-s|$

Aufgabe 4.6

- (a) $\sqrt{x-2}$; $D = \{x \in \mathbb{R} : x \geq 2\}$
- (b) $\sqrt{x+7}$; $D = \{x \in \mathbb{R} : x \geq -7\}$
- (c) $\sqrt{3-x}$; $D = \{x \in \mathbb{R} : x \leq 3\}$
- (d) $\sqrt{2x-3}$; $D = \{x \in \mathbb{R} : x \geq 1.5\}$

Aufgabe 4.7

- (a) $\sqrt{x^2-9}$; $D = \{x \in \mathbb{R} : 0 \leq x \leq 3\}$
- (b) $\sqrt{x^2+1}$; $D = \{x \in \mathbb{R} : -\infty < x < \infty\} = \mathbb{R}$
- (c) $\sqrt{4-x^2}$; $D = \{x \in \mathbb{R} : -2 \leq x \leq 2\}$
- (d) $\sqrt{x^2-10x+25} = \sqrt{(x-5)^2}$;
 $D = \{x \in \mathbb{R} : -\infty < x < \infty\} = \mathbb{R}$

Aufgabe 5.1

$$x\sqrt{3} + \sqrt{12} = \sqrt{32} - x\sqrt{3}$$

$$x\sqrt{3} + x\sqrt{3} = \sqrt{32} - \sqrt{12}$$

$$2x\sqrt{3} = 4\sqrt{2} - 2\sqrt{3}$$

$$x\sqrt{3} = 2\sqrt{2} - \sqrt{3}$$

$$x = \frac{2\sqrt{2} - \sqrt{3}}{\sqrt{3}}$$

$$x = \frac{(2\sqrt{2} - \sqrt{3}) \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}}$$

$$x = \frac{2\sqrt{6} - 3}{3}$$

$$x = -1 + \frac{2}{3}\sqrt{6}$$

Aufgabe 5.2

$$x\sqrt{2} - \sqrt{5} = \sqrt{20} + x$$

$$x\sqrt{2} - x = \sqrt{5} + 2\sqrt{5}$$

$$x(\sqrt{2} - 1) = 3\sqrt{5}$$

$$x = \frac{3\sqrt{5}}{(\sqrt{2} - 1)}$$

$$x = \frac{3\sqrt{5} \cdot (\sqrt{2} + 1)}{(\sqrt{2} - 1)(\sqrt{2} + 1)}$$

$$x = \frac{3\sqrt{10} + 3\sqrt{5}}{2 - 1}$$

$$x = 3\sqrt{5} + 3\sqrt{10}$$

Aufgabe 5.3

$$(\sqrt{2} - x)^2 = (3 - x)(4 - x)$$

$$2 - 2x\sqrt{2} + x^2 = 12 - 7x + x^2$$

$$2 - 2x\sqrt{2} = 12 - 7x$$

$$7x - 2x\sqrt{2} = 10$$

$$x(7 - 2\sqrt{2}) = 10$$

$$x = \frac{10}{7 - 2\sqrt{2}}$$

$$x = \frac{10(7 + 2\sqrt{2})}{(7 - 2\sqrt{2})(7 + 2\sqrt{2})} = \frac{70 + 20\sqrt{2}}{49 - 8}$$

$$x = \frac{70}{41} + \frac{20}{41}\sqrt{2}$$

Aufgabe 6.1

(a) $x^2 = 289$

$$x_1 = 17$$

$$x_2 = -17$$

(b) $x^2 = \frac{49}{64}$

$$x_1 = \frac{7}{8}$$

$$x_2 = -\frac{7}{8}$$

$$(c) \quad x^2 = 4.41$$

$$x_1 = 2.1$$

$$x_2 = -2.1$$

Aufgabe 6.2

$$(a) \quad 2x^2 = 50$$

$$x^2 = 25$$

$$x = \pm 5$$

$$(b) \quad \frac{3}{2}x^2 = \frac{8}{27} \quad || \cdot \frac{2}{3}$$

$$x^2 = \frac{16}{81}$$

$$x = \pm \frac{4}{9}$$

$$(c) \quad 0.0002x^2 = 0.0098$$

$$x^2 = \frac{0.0098}{0.0002} = \frac{98}{2} = 49$$

$$x_2 = \pm 7$$

Aufgabe 6.3

$$(a) \quad (x+4)(x-4) = 9$$

$$x^2 - 16 = 9$$

$$x^2 = 25$$

$$x = \pm 5$$

$$(b) \quad (3x-5)(3x+5) = 24$$

$$9x^2 - 25 = 24$$

$$9x^2 = 49$$

$$x^2 = \frac{49}{9}$$

$$x = \pm \frac{7}{3}$$

Aufgabe 6.4

$$(a) \quad (x-7)^2 = 121 = 11^2$$

$$x-7 = 11 \quad \vee \quad x-7 = -11$$

$$x_1 = 18 \quad \quad \quad x_2 = -4$$

$$(b) (2x + 3)^2 = 529 = 23^2$$

$$2x + 3 = 23 \quad \vee \quad 2x + 3 = -23$$

$$2x = 20 \qquad \qquad 2x = -26$$

$$x_1 = 10 \qquad \qquad x_2 = -13$$

Aufgabe 6.5

$$(2x - 11)^2 = (x - 10)^2$$

$$2x - 11 = x - 10 \quad \vee \quad 2x - 11 = -(x - 10)$$

$$x = 1 \qquad \qquad 2x - 11 = -x + 10$$

$$3x = 21$$

$$x = 7$$

Aufgabe 6.6

$$(2x + 5)^2 = (4x - 7)^2$$

$$2x + 5 = 4x - 7 \quad \vee \quad 2x + 5 = -(4x - 7)$$

$$-2x = -12 \qquad \qquad 2x + 5 = -4x + 7$$

$$x = 6 \qquad \qquad 6x = 2$$

$$x = \frac{1}{3}$$

Aufgabe 6.7

$$a = x, b = 2x \text{ und } c = 5x$$

$$S = 2(ab + bc + ca)$$

$$1.36 = 2(2x^2 + 10x^2 + 5x)$$

$$0.68 = 2x^2 + 10x^2 + 5x$$

$$0.68 = 17x^2$$

$$x^2 = 0.04$$

$$x = 0.2 \text{ dm}$$

Kantenlängen: 2 cm, 4 cm, 10 cm

Aufgabe 7.1

$$\sqrt{x - 1} = 3 \quad ||^2$$

$$x - 1 = 9 \quad || + 1$$

$$x = 10$$

$$L: \sqrt{10 - 1} = \sqrt{9} = 3$$

R: 3

$$L = \{10\}$$

Aufgabe 7.2

$$\begin{aligned}\sqrt{x+6} &= 2\sqrt{x} \quad ||^2 \\ x+6 &= 4x \quad || -x \\ 6 &= 3x \quad || :2 \\ 2 &= x\end{aligned}$$

$$L: \sqrt{2+6} = \sqrt{8}$$

$$R: 2\sqrt{2} = \sqrt{4} \cdot \sqrt{2} = \sqrt{4 \cdot 2} = \sqrt{8}$$

$$L = \{2\}$$

Aufgabe 7.3

$$\begin{aligned}\sqrt{x+6} &= \sqrt{2x} \quad ||^2 \\ x+6 &= 2x \quad || -x \\ 6 &= x\end{aligned}$$

$$L: \sqrt{6+6} = \sqrt{12}$$

$$R: \sqrt{2 \cdot 6} = \sqrt{12}$$

$$L = \{6\}$$

Aufgabe 7.4

$$\begin{aligned}\sqrt{x+1} &= x-1 \quad ||^2 \\ x+1 &= x^2 - 2x + 1 \quad || -x - 1 \\ 0 &= x^2 - 3x \\ 0 &= x(x-3) \\ x_1 &= 0 \\ x_2 &= 3\end{aligned}$$

$$x = 0; L: \sqrt{0+1} = \sqrt{1} = 1; R: 0 - 1 = -1$$

$$x = 3; \text{rhs: } \sqrt{3+1} = \sqrt{4} = 2; \text{lhs: } 3 - 1 = 2$$

$$L = \{3\}$$

Aufgabe 7.5

$$\begin{aligned}
x + 2\sqrt{x-2} &= 1 \quad || -x \\
2\sqrt{x-2} &= 1 - x \quad ||^2 \\
4(x-2) &= 1 - 2x + x^2 \\
4x - 8 &= 1 - 2x + x^2 \\
0 &= x^2 - 6x + 9 \\
0 &= (x-3)^2 \\
x &= 3
\end{aligned}$$

$$L: 3 + 2\sqrt{3-2} = 3 + 2\sqrt{1} = 3 + 2 \cdot 1 = 5$$

R: 1

$$L = \{ \}$$

Aufgabe 7.6

$$\begin{aligned}
\sqrt{x^2-4} - x &= 1 \quad || +x \\
\sqrt{x^2-4} &= x+1 \quad ||^2 \\
x^2-4 &= x^2+2x+1 \quad || -x^2-1 \\
-5 &= 2x \quad || :2 \\
-2.5 &= x
\end{aligned}$$

$$L: \sqrt{6.25-4} - (-2.5) = \sqrt{2.25} + 2.5 = 1.5 + 2.5 = 4$$

R: 1

$$L = \{ \}$$

Aufgabe 7.7

$$\begin{aligned}
\sqrt{x-7} - 1 &= \sqrt{x-8} \quad ||^2 \\
(\sqrt{x-7} - 1)^2 &= x-8 \quad \text{BF2} \\
x-7-2\sqrt{x-7}+1 &= x-8 \quad || +2\sqrt{x-7}-x+8 \\
2 &= 2\sqrt{x-7} \quad || :2 \\
1 &= \sqrt{x-7} \quad ||^2 \\
x-7 &= 1 \\
x &= 8
\end{aligned}$$

$$L: \sqrt{8-7} - 1 = \sqrt{1} - 1 = 1 - 1 = 0$$

$$R: \sqrt{8-8} = \sqrt{0} = 0$$

$$L = \{8\}$$

Aufgabe 7.8

$$\begin{aligned}\sqrt{x} + 1 &= \sqrt{x + 11} \quad ||^2 \\ x + 2\sqrt{x} + 1 &= x + 11 \quad || -x - 1 \\ 2\sqrt{x} &= 10 \quad || : 2 \\ \sqrt{x} &= 5 \quad ||^2 \\ x &= 25\end{aligned}$$

$$L: \sqrt{25} + 1 = 5 + 1 = 6$$

$$R: \sqrt{25 + 11} = \sqrt{36} = 6$$

$$L = \{25\}$$

Aufgabe 7.9

$$\begin{aligned}\sqrt{\sqrt{x+4} - x} &= 2 \quad ||^2 \\ \sqrt{x+4} - x &= 4 \quad || + x \\ \sqrt{x+4} &= x + 4 \quad ||^2 \\ x + 4 &= x^2 + 8x + 16 \\ 0 &= x^2 + 7x + 12 \\ 0 &= (x + 3)(x + 4) \\ x_1 &= -3 \\ x_2 &= -4\end{aligned}$$

$$x = -3; \text{ lhs: } \sqrt{\sqrt{-3+4} - (-3)} = \sqrt{1+3} = 2; \text{ rhs: } 2$$

$$x = -4; \text{ lhs: } \sqrt{\sqrt{-4+4} - (-4)} = \sqrt{0+4} = 2; \text{ rhs: } 2$$

$$L = \{-3, -4\}$$