

**Aufgabe 1.1**

(a)  $\sqrt{196} = 14$

(e)  $\sqrt{361} = 19$

(b)  $\sqrt{121} = 11$

(f)  $\sqrt{625} = 25$

(c)  $\sqrt{441} = 21$

(g)  $\sqrt{225} = 15$

(d)  $\sqrt{144} = 12$

(h)  $\sqrt{484} = 22$

**Aufgabe 1.2**

(a)  $\sqrt{887503681}$

Vorkommastellen:  $\lceil 9/2 \rceil = \lceil 4.5 \rceil = 5$

Nachkommastellen: keine

(b)  $\sqrt{20510.536225}$

Vorkommastellen:  $\lceil 5/2 \rceil = \lceil 2.5 \rceil = 3$

Nachkommastellen:  $6/2 = 3$

**Aufgabe 2.1**

(a)  $\sqrt{0.000324} = 0.018$

(b)  $\sqrt{3.61} = 1.9$

(c)  $\sqrt{16.9 \cdot 10^9} = 1.3 \cdot 10^4 = 130\,000$

**Aufgabe 2.2**

(a)  $\sqrt{6}\sqrt{15}\sqrt{5}\sqrt{2} = \sqrt{2}\sqrt{3}\sqrt{3}\sqrt{5}\sqrt{5}\sqrt{2} = 2 \cdot 3 \cdot 5 = 30$

(b)  $\sqrt{225^2} = 225$

(c)  $\sqrt{17^4} = 17^2 = 289$

(d)  $\sqrt{(-35)^2} = 35$

(e)  $\sqrt{8.1 \cdot 10^9} = \sqrt{81 \cdot 10^8} = 9 \cdot 10^4$

(f)  $\sqrt{0.144 \cdot 10^7} = \sqrt{1.44 \cdot 10^6} = 1.2 \cdot 10^3$

(g)  $\sqrt{0.00043} \cdot \sqrt{0.043} = \sqrt{0.0043} \cdot \sqrt{0.0043} = 0.0043$

(h)  $\sqrt{0.0156} \cdot \sqrt{15\,600} = \sqrt{15.6} \cdot \sqrt{15.6} = 15.6$

### Aufgabe 2.3

$$(a) \frac{\sqrt{28}}{\sqrt{7}} = \sqrt{\frac{28}{7}} = \sqrt{4} = 2$$

$$(b) \frac{\sqrt{3240}}{\sqrt{10}} = \sqrt{\frac{3240}{10}} = \sqrt{324} = 18$$

$$(c) \sqrt{\frac{7}{3}} \sqrt{\frac{5}{6}} \sqrt{\frac{7}{5}} = \sqrt{\frac{7}{3} \cdot \frac{5}{6} \cdot \frac{7}{5}} = \sqrt{\frac{49}{36}} = \frac{7}{6}$$

### Aufgabe 2.4

$$(a) 2\sqrt{3} = \sqrt{4}\sqrt{3} = \sqrt{12}$$

$$(b) 3\sqrt{2} = \sqrt{9}\sqrt{2} = \sqrt{18}$$

$$(c) 5\sqrt{5} = \sqrt{25}\sqrt{5} = \sqrt{125}$$

$$(d) 10\sqrt{7} = \sqrt{100}\sqrt{7} = \sqrt{700}$$

### Aufgabe 2.5

$$(a) \sqrt{441\,000\,000} = \sqrt{441} \cdot \sqrt{1\,000\,000} = 21 \cdot 1000 = 21\,000$$

$$(b) \sqrt{0.000064} = 0.008$$

$$(c) \sqrt{11^{18}} = 11^9$$

$$(d) \sqrt{6\frac{1}{4}} = \sqrt{\frac{25}{4}} = \frac{\sqrt{25}}{\sqrt{4}} = \frac{5}{2}$$

$$(e) \sqrt{\sqrt{\sqrt{256}}} = \sqrt{\sqrt{16}} = \sqrt{4} = 2$$

$$(f) \sqrt{19 + \sqrt{29 + \sqrt{49}}} = \sqrt{19 + \sqrt{36}} = \sqrt{25} = 5$$

### Aufgabe 3.1

$$(a) \sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$$

$$(b) \sqrt{9000} = \sqrt{900 \cdot 10} = 30\sqrt{10}$$

$$(c) \sqrt{75} = \sqrt{25 \cdot 3} = 5\sqrt{3}$$

$$(d) \sqrt{7^{23}} = \sqrt{7^{22} \cdot 7} = \sqrt{7^{22}} \cdot \sqrt{7} = 7^{11} \cdot \sqrt{7}$$

### Aufgabe 3.2

$$(a) \sqrt{50} + \sqrt{32} - \sqrt{75} + \sqrt{150} = 5\sqrt{2} + 4\sqrt{2} - 5\sqrt{3} + 5\sqrt{6} = 9\sqrt{2} - 5\sqrt{3} + 5\sqrt{6}$$

$$(b) (7 + \sqrt{3})(7 - \sqrt{3}) = 49 - 3 = 46$$

$$(c) (4 + \sqrt{2})^2 = 16 + 2 \cdot 4 \cdot \sqrt{2} + 2 = 18 + 8\sqrt{2}$$

$$(d) (\sqrt{3} + \sqrt{5})^2 = 3 + 2\sqrt{3}\sqrt{5} + 5 = 8 + 2\sqrt{15}$$

### Aufgabe 3.3

$$(a) (5 - \sqrt{7})^2 = 25 - 10\sqrt{7} - 7 = 18 - 10\sqrt{7}$$

(b)  $\sqrt{4\sqrt{3}}$  kann nicht in die Normalform gebracht werden.

$$(c) \left( \sqrt{3 - \sqrt{2}} + \sqrt{3 + \sqrt{2}} \right)^2 \\ = (3 - \sqrt{2}) + 2\sqrt{3 - \sqrt{2}}\sqrt{3 + \sqrt{2}} + (3 + \sqrt{2}) \\ = 6 + 2\sqrt{(3 - \sqrt{2})(3 + \sqrt{2})} = 6 + 2\sqrt{9 - 2} = 6 + 2\sqrt{7}$$

### Aufgabe 3.4

$$(a) \frac{2}{\sqrt{8}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{1 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{1}{2}\sqrt{2}$$

$$(b) \frac{\sqrt{3}}{\sqrt{3} - \sqrt{2}} = \frac{\sqrt{3} \cdot (\sqrt{3} + \sqrt{2})}{(\sqrt{3} - \sqrt{2}) \cdot (\sqrt{3} + \sqrt{2})} \\ = \frac{3 + \sqrt{6}}{3 - 2} = 3 + \sqrt{6}$$

$$(c) \frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} + \sqrt{5}} = \frac{(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{5})}{(\sqrt{2} + \sqrt{5})(\sqrt{2} - \sqrt{5})} \\ = \frac{2 - \sqrt{10} + \sqrt{6} - \sqrt{15}}{2 - 5} \\ = -\frac{2}{3} - \frac{1}{3}\sqrt{6} + \frac{1}{3}\sqrt{10} + \frac{1}{3}\sqrt{15}$$

### Aufgabe 4.1

(a)  $\sqrt{b^2} = |b|$

(b)  $\sqrt{a^2 + a^2 + a^2 + a^2} = \sqrt{4a^2} = 2|a|$

(c)  $\sqrt{9b^4c^2} = 3b^2|c|$

(d)  $\sqrt{4x^2 - 12xy + 9y^2} = \sqrt{(2x - 3y)^2} = |2x - 3y|$

(e)  $\sqrt{(x - 3)^2 + 6x} = \sqrt{x^2 - 6x + 9 - 12x} = \sqrt{x^2 + 6x + 9} = \sqrt{(x + 3)^2} = |x + 3|$

(f)  $(\sqrt{x^3})^2 = x^3$

### Aufgabe 4.2

(a)  $\sqrt{\frac{x^4}{y^2}} = \frac{\sqrt{x^4}}{\sqrt{y^2}} = \frac{x^2}{|y|}$

(b)  $\sqrt{\frac{1}{t}\sqrt{t^3}} \cdot \sqrt{t^3\sqrt{t}} = \sqrt{\frac{1}{t}\sqrt{t^3} \cdot t^3\sqrt{t}} = \sqrt{t^2\sqrt{t^4}}$   
 $= \sqrt{t^2 \cdot t^2} = t^2$

### Aufgabe 5.1

$$\sqrt{3}x = x + 1$$

$$\sqrt{3}x - x = 1$$

$$x(\sqrt{3} - 1) = 1$$

$$x = \frac{1}{\sqrt{3} - 1}$$

$$x = \frac{1 \cdot (\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$$

$$x = \frac{1 + \sqrt{3}}{3 - 1}$$

$$x = \frac{1}{2} + \frac{1}{2}\sqrt{3}$$

### Aufgabe 5.2

$$3x - \sqrt{7} = \sqrt{5}x + 4$$

$$3x - \sqrt{5}x = 4 + \sqrt{7}$$

$$x(3 - \sqrt{5}) = 4 + \sqrt{7}$$

$$x = \frac{4 + \sqrt{7}}{3 - \sqrt{5}}$$

$$x = \frac{(4 + \sqrt{7})(3 + \sqrt{5})}{(3 - \sqrt{5})(3 + \sqrt{5})}$$

$$x = \frac{12 + 4\sqrt{5} + 3\sqrt{7} + \sqrt{35}}{9 - 5}$$

$$= 3 + \sqrt{5} + \frac{3}{4}\sqrt{7} + \frac{1}{4}\sqrt{35}$$

### Aufgabe 6.1

$$3x^2 = 0.48$$

$$x^2 = 0.16$$

$$x = \pm 0.4$$

$$L = \{\pm 0.4\}$$

### Aufgabe 6.2

$$(x + 9)^2 = 4x^2$$

$$x + 9 = 2x \quad x + 9 = -2x$$

$$9 = x \quad 3x = -9$$

$$x = 9 \quad x = -3$$

$$L = \{9, -3\}$$

### Aufgabe 6.3

$$(4x + 3)^2 = (3x - 10)^2$$

$$4x + 3 = +(3x - 10) \quad 4x + 3 = -(3x - 10)$$

$$4x + 3 = 3x - 10 \quad 4x + 3 = -3x + 10$$

$$x = -13 \quad 7x = 7$$

$$x = 1$$

$$L = \{-13, 1\}$$

### Aufgabe 7.1

$$\sqrt{2x+4} = \sqrt{8-x} \quad ||^2$$

$$2x+4 = 8-x$$

$$3x = 4$$

$$x = \frac{4}{3}$$

Probe:

$$\text{L: } \sqrt{2 \cdot \frac{4}{3} + 4} = \sqrt{\frac{8}{3} + \frac{12}{3}} = \sqrt{\frac{20}{3}}$$

$$\text{R: } \sqrt{8 - \frac{4}{3}} = \sqrt{\frac{24}{3} - \frac{4}{3}} = \sqrt{\frac{20}{3}} \text{ (stimmt)}$$

$$L = \left\{ \frac{4}{3} \right\}$$

### Aufgabe 7.2

$$\sqrt{x} + 4 = 7\sqrt{x} \quad || -\sqrt{x}$$

$$4 = 6\sqrt{x} \quad || : 2$$

$$2 = 3\sqrt{x} \quad ||^2$$

$$4 = 9x \quad || : 9$$

$$x = \frac{4}{9}$$

Probe:

$$\text{L: } \sqrt{\frac{4}{9}} + 4 = \frac{2}{3} + \frac{12}{3} = \frac{14}{3}$$

$$\text{R: } 7\sqrt{\frac{4}{9}} = 7\frac{2}{3} = \frac{14}{3} \text{ (stimmt)}$$

$$L = \left\{ \frac{4}{9} \right\}$$

### Aufgabe 7.3

$$\sqrt{x^2-2} = x-1 \quad ||^2$$

$$x^2-2 = x^2-2x+1 \quad || -x^2-1$$

$$-3 = -2x \quad || : (-2)$$

$$x = \frac{3}{2}$$

Probe:

$$\text{L: } \sqrt{\left(\frac{3}{2}\right)^2 - 2} = \sqrt{\frac{9}{4} - \frac{8}{4}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$\text{R: } \frac{3}{2} - 1 = \frac{1}{2} \text{ (stimmt)}$$

$$L = \left\{ \frac{3}{2} \right\}$$

#### Aufgabe 7.4

$$\sqrt{x} + \sqrt{3} = \sqrt{x+15}$$

$$(\sqrt{x} + \sqrt{3})^2 = \sqrt{x+15}^2 \quad (\text{Doppelprodukt!})$$

$$x + 2\sqrt{x}\sqrt{3} + 3 = x + 15 \quad || -x - 3$$

$$2\sqrt{3x} = 12$$

$$\sqrt{3x} = 6$$

$$3x = 36$$

$$x = 12$$

Probe:

$$\text{L: } \sqrt{12} + \sqrt{3} = 2\sqrt{3} + \sqrt{3} = 3\sqrt{3}$$

$$\text{R: } \sqrt{12+15} = \sqrt{27} = 3\sqrt{3}$$

$$L = \{12\}$$

#### Aufgabe 7.5

$$\sqrt{x+5} + \sqrt{x} = \sqrt{5x+5}$$

$$x + 5 + 2\sqrt{x+5}\sqrt{x} + x = 5x + 5$$

$$2\sqrt{x+5}\sqrt{x} = 3x$$

$$4(x+5)x = 9x^2$$

$$4x^2 + 20x = 9x^2$$

$$0 = 5x^2 - 20x$$

$$0 = 5x(x-4)$$

$$x_1 = 0$$

$$x_2 = 4$$

Probe für  $x = 0$

$$\text{L: } \sqrt{0+5} + \sqrt{0} = \sqrt{5}$$

$$\text{R: } \sqrt{5 \cdot 0 + 5} = \sqrt{5} \quad (\text{ok})$$

Probe für  $x = 4$

$$\text{L: } \sqrt{4+5} + \sqrt{4} = 5$$

$$\text{R: } \sqrt{5 \cdot 4 + 5} = 5 \quad (\text{ok})$$

$$L = \{0, 4\}$$