

Aufgabe 8.1

(a) $x^8 + x^3 = x^3(x^5 + 1)$

(b) $x^7 - x^5 = x^5(x^2 - 1) = x^5(x + 1)(x - 1)$

Aufgabe 8.2

(a) $x^4 - y^4 = (x^2 + y^2)(x^2 - y^2) = (x^2 + y^2)(x + y)(x - y)$

(b) $nx^2 - ny^2 = n(x^2 - y^2) = n(x + y)(x - y)$

Aufgabe 8.3

(a) $x^4 - 4x^3 + 4x^2 = x^2(x^2 - 4x + 4) = x^2(x - 2)^2$

(b) $x^9 - 2x^8 - 15x^7 = x^7(x^2 - 2x - 15) = x^7(x + 3)(x - 5)$

Aufgabe 8.4

(a) $x^{n+2} - x^n = x^n(x^2 - 1) = x^n(x + 1)(x - 1)$

(b) $x^{2n+3} + 2x^{2n+2} + x^{2n+1} = x^{2n+1}(x^2 + 2x + 1) = x^{2n+1}(x + 1)^2$

Aufgabe 8.5

$$\begin{aligned} \text{(a)} \quad \frac{16^{13} \cdot 18^8 \cdot 24^3}{48^{17}} &= \frac{(2^4)^{13} \cdot (2 \cdot 3^2)^8 \cdot (2^3 \cdot 3^1)^3}{(2^4 \cdot 3)^{17}} \\ &= \frac{2^{52} \cdot 2^8 \cdot 3^{16} \cdot 2^9 \cdot 3^3}{2^{68} \cdot 3^{17}} \\ &= \frac{2^{69} \cdot 3^{19}}{2^{68} \cdot 3^{17}} = 2 \cdot 3^2 = 18 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \frac{27^{12} \cdot 16^7 \cdot 12^{13}}{36^{24}} &= \frac{(3^3)^{12} \cdot (2^4)^7 \cdot (2^2 \cdot 3)^{13}}{(2^2 \cdot 3^2)^{24}} \\ &= \frac{3^{36} \cdot 2^{28} \cdot 2^{26} \cdot 3^{13}}{2^{48} \cdot 3^{48}} \\ &= \frac{2^{54} \cdot 3^{49}}{2^{48} \cdot 3^{48}} = 2^6 \cdot 3 = 192 \end{aligned}$$

Aufgabe 8.6

$$\text{(a)} \quad \frac{x^{10} + x^8}{x^5} = \frac{x^8(x^2 + 1)}{x^5} = x^3(x^2 + 1)$$

$$\text{(b)} \quad \frac{x^9 - x^7}{x^5 - x^4} = \frac{x^7(x^2 - 1)}{x^4(x - 1)} = \frac{x^3(x - 1)(x + 1)}{x - 1} = x^3(x + 1)$$

Aufgabe 8.7

$$\text{(a)} \quad \frac{x^8 + x^7}{x^4 + x^3} = \frac{x^7(x + 1)}{x^3(x + 1)} = x^4$$

$$\begin{aligned} \text{(b)} \quad \frac{x^{12} - 1}{x^3 - 1} &= \frac{(x^6 + 1)(x^6 - 1)}{x^3 - 1} = \frac{(x^6 + 1)(x^3 + 1)(x^3 - 1)}{x^3 - 1} \\ &= (x^6 + 1)(x^3 + 1) \end{aligned}$$