Aufgabe 7.1

(a) $x^4 = 81$
$x^4 = 3^4$
$x = \pm 3$

(b) $2x^3 = 250$
$x^3 = 125 = 5^3$
$x = 5$

Aufgabe 7.2

(a) $x^8 = 16$
$x^8 = 2^4 = \sqrt{2}^8$
$x = \pm\sqrt{2}$

(b) $x^5 = 49\sqrt{7} = 7^2\sqrt{7} = \sqrt{7}^4 \cdot \sqrt{7}^1$
$x^5 = \sqrt{7}^5$
$x = \sqrt{7}$

Aufgabe 7.3

(a) $x^5 = -243 = -3^5$
$x^5 = (-3)^5$
$x = -3$

(b) $x^4 = -625$ nicht lösbar in $\mathbb{R}$

Aufgabe 7.4

(a) $x^5 = x^4$
$x^5 - x^4 = 0$
$x^4(x-1) = 0$
$x_1 = 0$
$x_2 = 1$

(b) $x^7 = 2x^6$
$x^7 - 2x^6 = 0$
$x^6(x-2) = 0$
$x_1 = 0$
$x_2 = 2$

Aufgabe 7.5

(a) $5^2 \cdot 5^5 = 5^{n+1}$

$$5^7 = 5^{n+1}$$
$$7 = n + 1$$
$$n = 6$$

(b) $7^9 : 7^{n-1} = 7^{2n-3} \cdot 7^4$

$$7^{10-n} = 7^{2n+1}$$
$$10 - n = 2n + 1$$
$$9 = 3n$$
$$n = 3$$

Aufgabe 7.6

(a) $3^4 \cdot 3^n = 3^2$

$$3^{4+n} = 2^2$$
$$4 + n = 2$$
$$n = -2$$

(b) $4 \cdot 2^{n+1} = 16^3$

$$2^2 \cdot 2^{n+1} = 16^3$$
$$2^{n+3} = 16^3$$
$$n + 3 = 3$$
$$n = 0$$

Aufgabe 7.7

(a) $8^{2n-2} = 4^{2n+4}$

$$(2^3)^{2n-2} = (2^2)^{2n+4}$$
$$2^{3(2n-2)} = 2^{2(2n+4)}$$
$$2^{6n-6} = 2^{4n+8}$$
$$6n - 6 = 4n + 8$$
$$2n = 14$$
$$n = 7$$

(b) $(10^n)^2 = (10^{10})^5$

$$10^{2n} = 10^{50}$$
$$2n = 50$$
$$n = 25$$

Aufgabe 7.8

$$
\begin{aligned}
25^n \cdot 5^{n+1} &= 125^{n+4} \\
(5^2)^n \cdot 5^{n+1} &= (5^3)^{n+4} \\
5^{2n} \cdot 5^{n+1} &= 5^{3(n+4)} \\
5^{2n+n+1} &= 5^{3n+12} \\
5^{3n+1} &= 5^{3n+12} \\
3n+1 &= 3n+12 \\
1 &= 12
\end{aligned}
$$

keine Lösung