Aufgabe 4.1

(a) $2^4 \cdot 5^4 = (2 \cdot 5)^4 = 10^4$

(b) $0.5^5 \cdot 6^5 = (0.5 \cdot 6)^5 = 3^5 = 243$

Aufgabe 4.2

(a) $\left(\frac{4}{15}\right)^4 \cdot \left(\frac{5}{2}\right)^4 = \left(\frac{4}{15} \cdot \frac{5}{2}\right)^4 = \left(\frac{2}{3}\right)^4 = \frac{16}{81}$

(b) $\left(-\frac{39}{7}\right)^5 \cdot \left(-\frac{7}{13}\right)^5 = \left(\frac{39}{7} \cdot \frac{7}{13}\right)^5 = 3^5 = 243$

Aufgabe 4.3

(a) $\left(\sqrt{2}\right)^3 \cdot \left(\sqrt{8}\right)^3 = \left(\sqrt{2} \cdot \sqrt{8}\right)^3 = \left(\sqrt{16}\right)^3 = 4^3 = 64$

(b) $0.25^{17} \cdot (-4)^{17} = \big(0.25 \cdot (-4)\big)^{17} = \big(-1\big)^{17} = -1$

Aufgabe 4.4

(a) $x^8 \cdot y^8 = (xy)^8$

(b) $4^p \cdot 5^p \cdot 2^p = 40^p$

Aufgabe 4.5

(a) $4^6 \cdot a^6 = (4a)^6$

(b) $r^{12} \cdot 5^{12} \cdot s^{12} = (5rs)^{12}$

Aufgabe 4.6

(a) $(3a)^9 \cdot (4b)^9 = (12ab)^9$

(b) $0.25^{15} \cdot y^{15} \cdot 12^{15} = (3y)^{15}$

Aufgabe 4.7

(a) $2^k \cdot x^k \cdot y^k = (2xy)^k$

(b) $3^{2k+1} \cdot (-a)^{2k+1} = (-3a)^{2k+1} = -3a^{2k+1}$

Aufgabe 4.8

$$\left(\frac{u}{v}\right)^n \cdot \left(\frac{v}{w}\right)^n \cdot \left(\frac{w}{u}\right)^n = \left(\frac{u}{v} \cdot \frac{v}{w} \cdot \frac{w}{u}\right)^n = 1^n = 1$$

Aufgabe 4.9

(a) $12^3 : 4^3 = (12 : 4)^3 = 3^3 = 27$

(b) $8^9 : 0.5^9 = (8 \cdot 0.5)^9 = 16^9$

Aufgabe 4.10

(a) $12^s : 2.4^s = (12 : 2.4)^s = 5^s$

(b) $0.52^{p+1} : 0.13^{p+1} = (0.52 : 0.13)^{p+1} = 4^{p+1}$

Aufgabe 4.11

(a) $\left(\frac{3}{2}\right)^7 : \left(\frac{3}{4}\right)^7 = \left(\frac{3}{2} : \frac{3}{4}\right)^7 = \left(\frac{3}{2} \cdot \frac{4}{3}\right)^7 = \left(\frac{1}{2}\right)^7 = \frac{1}{128}$

(b) $\left(\frac{15}{4}\right)^m : \left(\frac{5}{12}\right)^m = \left(\frac{15}{4} : \frac{5}{12}\right)^m = \left(\frac{15}{4} \cdot \frac{12}{5}\right)^m = 9^m$

Aufgabe 4.12

(a) $3^8 : \left(\sqrt{3}\right)^8 = \left(\sqrt{9} : \sqrt{3}\right)^8 = \left(\sqrt{3}\right)^8 = 3^4 = 81$

(b) $\left(\sqrt{45}\right)^5 : \left(\sqrt{5}\right)^5 = \left(\sqrt{45} : \sqrt{5}\right)^5 = \left(\sqrt{9}\right)^5 = 3^5 = 243$

Aufgabe 4.13

(a) $(2x)^n : x^n = (2x : x)^n = 2^n$

(b) $(pq)^k : p^k = (pq : p)^k = q^k$

Aufgabe 4.14

$(6abc)^{n+1} : (2b)^{n+1} : c^{n+1} = (6abc : 2b : c)^{n+1} = (3a)^{n+1}$

Aufgabe 4.15

$$\begin{aligned}\left(\frac{b}{d}\right)^{2n} : \left(-\frac{3}{d}\right)^{2n} : \left(\frac{b}{5}\right)^{2n} &= \left(-\frac{b}{d} : \frac{3}{d} : \frac{b}{5}\right)^{2n} \\ &= \left(-\frac{b}{d} \cdot \frac{d}{3} \cdot \frac{5}{b}\right)^{2n} \\ &= \left(-\frac{5}{3}\right)^{2n} = \left(\frac{5}{3}\right)^{2n}\end{aligned}$$