Aufgabe 1

(a) $9^3 = 729$ (b) $3^5 = 243$ (c) $7^3 = 343$ (d) $2^8 = 256$

Aufgabe 2

(a) $5^3 = 125$ (b) $2^{10} = 1024$ (c) $3^4 = 81$ (d) $6^3 = 216$

Aufgabe 3

(a) $17^0 = 1$ (b) $(-43)^0 = 1$ (c) $0^{17} = 0$ (d) 0^{-43} nicht def.

Aufgabe 4

(a) $\left(\frac{2}{7}\right)^3 = \frac{2^3}{7^3} = \frac{8}{343}$

(b) $\left(\frac{p}{q}\right)^n = \frac{p^n}{q^n}$

Aufgabe 5

(a) $(-2)^4 = 16$ (b) $(-2)^3 = -8$

Aufgabe 6

(a) $\sqrt{7}^6 = \sqrt{7} \cdot \sqrt{7} \cdot \sqrt{7} \cdot \sqrt{7} \cdot \sqrt{7} \cdot \sqrt{7} = 7^3 = 343$

(b) $\left(2 \cdot \sqrt{2}\right)^4 = 2^4 \cdot \sqrt{2}^4 = 16 \cdot 2^2 = 64$

(c) $\sqrt{3}^5 = \sqrt{3}^4 \cdot \sqrt{3}^1 = 3^2 \cdot \sqrt{3} = 9\sqrt{3}$

Aufgabe 7

(a) $10^{18} = 10^{3 \cdot 6} = 1$ Trillion

(b) $10^{15} = 10^{2 \cdot 6 + 3} = 1$ Billiarde

Aufgabe 8

(a) $0.0023 = 2.3 \cdot 10^{-3}$

(b) $54300 = 5.43 \cdot 10^{4}$

Aufgabe 9

(a) $a^{2n+3} \cdot a^{5n-7} = a^{2n+3+5n-7} = a^{7n-4}$

(b) $a^{3n-1} : a^{2n-4} = a^{3n-1-(2n-4)} = a^{3n-1-2n+4} = a^{n+3}$

(c) $\left(\frac{v}{w}\right)^n \cdot \left(\frac{w}{v}\right)^n = \left(\frac{v}{w} \cdot \frac{w}{v}\right)^n = 1^n = 1$

(d) $(8a^2)^n : (4a)^n = (8a^2 : 4a)^n = (2a)^n$

(e) $\left(a^{-3}\right)^{-2} = a^{(-3)\cdot(-2)} = a^6$

Aufgabe 10

(a) $c^7 + c^5 = c^5\left(c^2 + 1\right)$

(b) $z^9 - z^7 = z^7\left(z^2 - 1\right) = z^7(z+1)(z-1)$

(c) $p^5 - 14p^4 + 49p^3 = p^3\left(p^2 - 14p + 49\right) = p^3(p-7)^2$

(d) $q^4 + 5q^3 - 6q^2 = q^2\left(q^2 + 5q - 6\right) = q^2(q+3)(q+2)$

Aufgabe 11

(a) $(5s + 4r)^3$
$= 1 \cdot (5s)^3 + 3 \cdot (5s)^2(4r)^1 + 3 \cdot (5s)^1(4r)^2 + 1 \cdot (4r)^3$
$= 125s^3 + 300rs^2 + 240r^2s + 64r^3$

(b) $(4t - u)^3$
$= 1 \cdot (4t)^3 - 3 \cdot (4t)^2u^1 + 3 \cdot (4t)^1u^2 - 1 \cdot u^3$
$= 64t^3 - 48t^2u + 12tu^2 - u^3$

(c) $\left(z + z^{-1}\right)^4$
$= z^4 + 4z^3\left(z^{-1}\right)^1 + 6z^2\left(z^{-1}\right)^2 + 4u^1\left(z^{-1}\right)^3 + \left(z^{-1}\right)^4$
$= z^4 + 4z^2 + 6 + 4z^{-2} + z^{-4}$

Aufgabe 12

(a) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

(b) $(-6)^{-2} = \frac{1}{(-6)^2} = \frac{1}{36}$

(c) $(-3)^{-3} = \frac{1}{(-3)^3} = \frac{1}{-27} = -\frac{1}{27}$

(d) $\left(\frac{1}{5}\right)^{-3} = \frac{5}{1}^{3} = 125$

(e) $\left(\frac{4}{3}\right)^{-2} = \left(\frac{3}{4}\right)^{2} = \frac{9}{16}$

Aufgabe 13

(c) $0.2 = \frac{1}{5} = 5^{-1}$

(b) $0.125 = \frac{1}{8} = \frac{1}{2^3} = 2^{-3}$

(c) $0.\overline{3} = \frac{1}{3} = 3^{-1}$

(d) $0.\overline{1} = \frac{1}{9} = \frac{1}{3^2} = 3^{-2}$

Aufgabe 14

$$\begin{aligned}
\left(\frac{a^{-3}b^2}{b^{-1}c^3}\right)^4 : \left(\frac{a^2c^{-5}}{a^3b^5}\right)^{-3} &= \left(a^{-3}b^2b^1c^{-3}\right)^4 \cdot \left(a^2c^{-5}a^{-3}b^{-5}\right)^3 \\
&= \left(a^{-3}b^3c^{-3}\right)^4 \cdot \left(a^{-1}b^{-5}c^{-5}\right)^3 \\
&= a^{-12}b^{12}c^{-12} \cdot a^{-3}b^{-15}c^{-15} \\
&= a^{-15}b^{-3}c^{-27}
\end{aligned}$$

Aufgabe 15

(a)
$$\begin{aligned}
2^{500} &> 3^{400} \\
\left(2^5\right)^{100} &> \left(3^4\right)^{100} \\
32^{100} &> 81^{100} \quad \text{falsch}
\end{aligned}$$

(b)
$$\begin{aligned}
25^{298} &< 125^{199} \\
\left(5^2\right)^{298} &> \left(5^3\right)^{199} \\
5^{596} &> 5^{597} \quad \text{wahr}
\end{aligned}$$

Aufgabe 16

(a)
$$\begin{aligned} x^5 &= 32 \\ x^5 &= 2^5 \\ x &= 2 \end{aligned}$$

(b)
$$\begin{aligned} x^6 &= 64 \\ x^6 &= 2^6 \\ x &= \pm 2 \end{aligned}$$

(c)
$$\begin{aligned} x^3 &= -216 \\ x^3 &= (-6)^3 \\ x &= -6 \end{aligned}$$

(d)
$$\begin{aligned} x^6 &= -64 \\ x^6 &= -2^6 \neq (-2)^6 \\ L &= \{\,\} \end{aligned}$$

(e)
$$\begin{aligned} x^6 &= 9x^5 \\ x^6 - 9x^5 &= 0 \\ x^5(x-9) &= 0 \\ x_1 &= 0 \\ x_2 &= 9 \end{aligned}$$

Aufgabe 17

(a)
$$\begin{aligned} 2^{16} : 2^{x-11} &= 2^{x+5} \cdot 2^4 \\ 2^{27-x} &= 2^{x+9} \\ 27 - x &= x + 9 \\ x &= 9 \end{aligned}$$

(b)
$$\begin{aligned} 100^{x+5} \cdot 10^{2x-3} &= 1000^{x-1} \\ 10^{2(x+5)} \cdot 10^{2x-3} &= 10^{3(x-1)} \\ 10^{4x+7} &= 10^{3x-3} \\ x &= -10 \end{aligned}$$