
Lineare Gleichungssysteme
Übungen (L+)

Version vom 10. März 2020

Aufgabe 1.1

- (a) $2u - 3v + 4/w = 8$ nein (wegen $1/w$)
- (b) $a_1 + 4a_2 + a_5 = -3$ ja
- (c) $5\sqrt{m} + 7\sqrt{n} - 8\sqrt{p} = 17$ nein (wegen \sqrt{m})
- (d) $\frac{1}{3}x - 15 = \frac{3}{5}y - 9$ ja ($\Rightarrow \frac{1}{3}x - \frac{3}{5}y = 6$)

Aufgabe 1.2

- (a) $5a - \sqrt{2}b + \pi c = 8; b - \sqrt{2}$
- (b) $u - v + w - x + y - z = 8; w - 1$
- (c) $-x_1 + 2x_2 - 4x_3 = -12; x_4 0$

Aufgabe 1.3

- (a) $x_1 + x_2 + x_4 = 1; (4, 2, 1, -5)$
 $1 \cdot 4 + 1 \cdot 2 + 0 \cdot 1 + 1 \cdot (-5) = 1 \quad \Rightarrow \quad \text{ja}$
- (b) $2s - 5t = -4; (8, 4)$
 $2 \cdot 8 - 5 \cdot 4 = -4 \quad \Rightarrow \quad \text{ja}$
- (c) $a + 3b - 4c + d = 3; (1, 0, 1, 0)$
 $1 \cdot 1 + 3 \cdot 0 - 4 \cdot 1 + 1 \cdot 0 = -3 \quad \Rightarrow \quad \text{nein}$
- (d) $\frac{1}{p} + \frac{1}{q} + \frac{1}{r} = 9; \left(\frac{1}{4}, \frac{1}{2}, \frac{1}{3}\right)$
 $4 + 2 + 3 = 9 \quad \Rightarrow \quad \text{ja}$
- (e) $x^2 + 2xy + y^2 + 4x = 0; (-1, 1)$
 $(-1)^2 + 2 \cdot (-1) \cdot 1 + 1^2 + 4 \cdot (-1) = -4 \quad \Rightarrow \quad \text{nein}$

Aufgabe 1.4

(a) $2e + 3f - 4g = 7; (3, 1, ?)$

$$6 + 3 - 4g = 7$$

$$-4g = -2$$

$$g = \frac{1}{2}$$

(b) $-y_1 + 5y_2 - 3y_4 = -8; (0, 2, ?, 3)$

$$0 + 10 + 0 \cdot y_3 - 12 = -8$$

$$-2 = -8 \quad \text{keine Lösung}$$

(c) $3u_1 - 2u_3 = 7; (1, ?, -2)$

$$3 - 0 \cdot u_2 - (-4) = 7$$

$$7 = 7 \quad \text{jedes } u_2 \in \mathbb{R} \text{ ist Lösung}$$

Aufgabe 1.5

(a) $4a - 7b = 3$

freie Variable: a

gebundene Variable: b

$$4a - 3 = 7b$$

$$b = \frac{4}{7}a - \frac{3}{7}$$

$$L = \left\{ \left(a, \frac{4}{7}a - \frac{3}{7} \right) : a \in \mathbb{R} \right\}$$

(b) $x_1 - 2x_2 + x_3 = 5$

freie Variable: x_1, x_2

gebundene Variable: x_3

$$x_3 = 5 - x_1 + 2x_2$$

$$L = \left\{ (x_1, x_2, 5 - x_1 + 2x_2) : x_1, x_2 \in \mathbb{R} \right\}$$

Aufgabe 1.6

$$(a) \quad \begin{array}{ll} 5x - 3y = 19 & 5 \cdot 5 - 3 \cdot 2 = 19 \\ -2x + 7y = 4 & -2 \cdot 5 + 7 \cdot 2 = 4 \end{array}$$

ja

$$(b) \quad \begin{array}{ll} x - 2y + 7z = 26 & -3 - 2 \cdot 2 + 7 \cdot 5 = 28 \neq 26 \\ 2x + 3y - 4z = -20 & \dots = \dots \end{array}$$

nein

$$(c) \quad \begin{array}{ll} x_1 + x_2 + x_3 + x_4 = 1 & 5 + 2 - 7 + 1 = 1 \\ x_1 - x_2 + x_3 + x_4 = -3 & 5 - 2 - 7 + 1 = -3 \\ x_1 + x_2 - x_3 + x_4 = 15 & 5 + 2 + 7 + 1 = 15 \\ x_1 + x_2 + x_3 - x_4 = 1 & 5 + 2 - 7 - 1 = -1 \neq 1 \end{array}$$

nein

Aufgabe 1.7

$$(a) \quad \begin{array}{l} 2x + 3y = 1 \\ 2x + 4y = 2 \end{array}$$

$$y = 1, x = -1 \quad \Rightarrow \quad L = \{(1, -1)\}$$

$$(b) \quad \begin{array}{l} x + y + z = 9 \\ x + y = 5 \\ y + z = 3 \end{array}$$

$$z = 4, x = 6, y = -1 \quad \Rightarrow \quad L = \{(6, -1, 4)\}$$

$$(c) \quad \begin{array}{l} x_1 + x_2 - x_3 = 1 \\ x_1 + x_2 - x_3 = 2 \\ x_1 + x_2 + x_3 = 3 \end{array}$$

$$L = \{ \}$$

$$(d) \quad \begin{array}{l} a + b + c + d = 15 \\ b + c + d = 17 \\ c + d = 8 \\ d = 3 \end{array}$$

$$d = 3, c = 5, b = 9, a = -2 \quad \Rightarrow \quad L = \{(-2, 9, 5, 3)\}$$

Aufgabe 1.8

(a) $y = 16 - 3x: L = \{(6, 2), (7, 5), (8, 8)\}$

(b) $7 = 4 - \frac{8}{7}x: L = \{\}$

(c) $y = 12 - \frac{9}{5}x: L = \{(5, 3)\}$

Aufgabe 1.9

(a) $L = \{(3, -2)\}$

(b) $L = \{(x, 3x + 5): x \in \mathbb{R}\}$

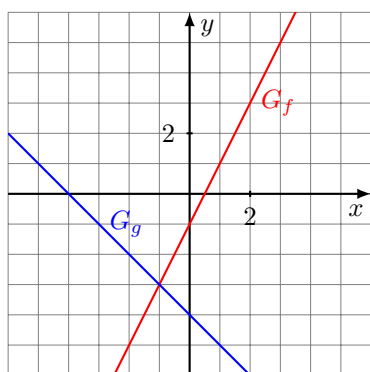
Aufgabe 1.10

(a) $L = \{(4, 4)\}$

(b) $L = \{(8, 8)\}$

Aufgabe 1.11

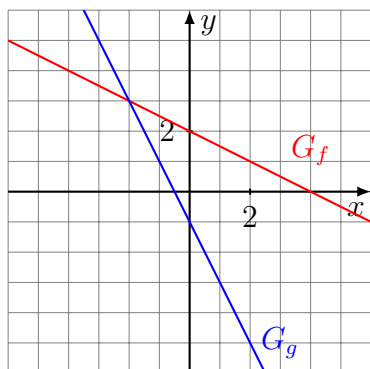
$$\begin{aligned} -2x + y = -1 &\Rightarrow f: y = 2x - 1 \\ x + y = -4 &\Rightarrow g: y = -x - 4 \end{aligned}$$



$$L = \{(-1, -3)\}$$

Aufgabe 1.12

$$\begin{aligned} x + 2y = 4 &\Rightarrow f: y = -\frac{1}{2}x + 2 \\ -6x - 3y = 3 &\Rightarrow g: y = -2x - 1 \end{aligned}$$



$$L = \{(-2, 3)\}$$

Aufgabe 2.1

$$4x + 3y = 14 \quad (1)$$

$$3x + y = 13 \quad (2)$$

$$(2) \text{ nach } y \text{ auflösen: } y = 13 - 3x \quad (3)$$

$$(3) \text{ in } (1) \text{ einsetzen: } 4x + 3(13 - 3x) = 14$$

$$4x + 39 - 9x = 14$$

$$-5x = -25$$

$$x = 5$$

$$x = 5 \text{ in } (3) \text{ einsetzen: } y = 13 - 3 \cdot 5 = -2$$

$$L = \{(5, -2)\}$$

Aufgabe 2.2

$$x = 2y - 1 \quad (1)$$

$$3x - 5y = 1 \quad (2)$$

$$(1) \text{ in } (2) \text{ einsetzen: } 3(2y - 1) - 5y = 1$$

$$6y - 3 - 5y = 1$$

$$y = 4$$

$$y = 4 \text{ in } (1) \text{ einsetzen: } x = 2 \cdot 4 - 1 = 7$$

$$L = \{(7, 4)\}$$

Aufgabe 2.3

$$4x + 2y = 20 \quad (1)$$

$$y = -3x \quad (2)$$

$$(2) \text{ in } (1) \text{ einsetzen: } 4x + 2 \cdot (-3x) = 20$$

$$4x - 6x = 20$$

$$-2x = 20$$

$$x = -10$$

$$x = -10 \text{ in } (2) \text{ einsetzen: } y = -3 \cdot (-10) = 30$$

$$L = \{(-10, 30)\}$$

Aufgabe 2.4

$$6x + 5y = -12 \quad (1)$$

$$2x = 4y + 13 \quad (2)$$

$$(2) \text{ in } (1) \text{ einsetzen: } 3 \cdot (4y + 13) + 5y = -12$$

$$12y + 39 + 5y = -12$$

$$17y = -51$$

$$y = -3$$

$$y = -3 \text{ in } (2) \text{ einsetzen: } 2x = 4 \cdot (-3) + 13 = 1$$

$$x = \frac{1}{2}$$

$$L = \left\{ \left(\frac{1}{2}, -3 \right) \right\}$$

Aufgabe 2.5

$$3x - 5y = 39 \quad (1)$$

$$5y = 17 - 4x \quad (2)$$

$$(2) \text{ in } (1) \text{ einsetzen: } 3x - (17 - 4x) = 39$$

$$3x - 17 + 4x = 39$$

$$7x = 56$$

$$x = 8$$

$$x = 8 \text{ in } (2) \text{ einsetzen: } 5y = 17 - 4 \cdot 8 = -15$$

$$y = -3$$

$$L = \{(8, -3)\}$$

Aufgabe 2.6

$$3x + 4y = 21 \quad (1)$$

$$6x + y = 0 \quad (2)$$

$$(2) \text{ nach } y \text{ auflösen: } y = -6x \quad (3)$$

$$(3) \text{ in } (1) \text{ einsetzen: } 3x + 4 \cdot (-6x) = 21$$

$$-21x = 21$$

$$x = -1$$

$$x = -1 \text{ in } (3) \text{ einsetzen: } y = -6 \cdot (-1) = 6$$

$$L = \{(-1, 6)\}$$

Aufgabe 2.7

$$8x + 13y = 79 \quad (1)$$

$$13y = 2x + 29 \quad (2)$$

(2) in (1) einsetzen: $8x + (2x + 29) = 79$

$$10x + 29 = 79$$

$$10x = 50$$

$$x = 5$$

$x = 5$ in (2) einsetzen: $13y = 2 \cdot 5 + 29$

$$13y = 39$$

$$y = 3$$

$$L = \{(5, 3)\}$$

Aufgabe 2.8

$$4(x + y) + 2y = 26 \quad (1)$$

$$2(x + y) = 3y - 15 \quad (2)$$

(2) in (1) einsetzen: $2(3y - 15) + 2y = 26$

$$8y - 30 = 26$$

$$8y = 56$$

$$y = 7$$

$y = 7$ in (2) einsetzen: $2(x + 7) = 21 - 15 = 6$

$$x + 7 = 3$$

$$x = -4$$

$$L = \{(-4, 7)\}$$

Aufgabe 2.9

$$4x - 5(x - y) = 12 \quad (1)$$

$$\frac{1}{2}(x - y) = 9 - 3x \quad (2)$$

(2) nach $(x - y)$ auflösen: $(x - y) = 18 - 6x \quad (3)$

(3) in (1) einsetzen: $4x - 5(18 - 6x) = 12$

$$4x - 90 + 30x = 12$$

$$34x = 102$$

$$x = 3$$

$x = 3$ in (3) einsetzen: $(3 - y) = 18 - 18 = 0$

$$y = 3$$

$$L = \{(3, 3)\}$$

Aufgabe 2.10

$$4x^2 - 3y = 4 \quad (1)$$

$$y = 2x^2 - 4 \quad (2)$$

(2) in (1) einsetzen: $4x^2 - 3(2x^2 - 4) = 4$

$$4x^2 - 6x^2 + 12 = 4$$

$$-2x^2 = -8$$

$$x^2 = 4$$

$$x_1 = 2 \quad \Rightarrow \quad y_1 = 4$$

$$x_2 = -2 \quad \Rightarrow \quad y_2 = 4$$

$$L = \{(2, 4), (-2, 4)\}$$

Aufgabe 2.11

$$2xy - 3x = 15 \quad (1)$$

$$xy + 7x = 33 \quad (2)$$

(2) nach xy auflösen: $xy = 33 - 7x$ (3)

(3) in (1) einsetzen: $2xy - 3x = 15$

$$2(33 - 7x) - 3x = 15$$

$$66 - 14x - 3x = 15$$

$$51 = 17x$$

$$x = 3$$

$x = 3$ in (2) einsetzen: $3y + 7 \cdot 3 = 33$

$$y = 4$$

$$L = \{(3, 4)\}$$

Aufgabe 2.12

$$\frac{(x+y)^2}{y} = 48 \quad (1)$$

$$x + y = 12 \quad (2)$$

(2) in (1) einsetzen: $\frac{12^2}{y} = 48$

$$12 \cdot 12 = 48y$$

$$12 = 4y$$

$$y = 3$$

$y = 3$ in (2) einsetzen: $x + 3 = 12$

$$x = 9$$

$$L = \{(9, 3)\}$$

Aufgabe 3.1

$$5x + y = 7 \quad (1)$$

$$-x + y = -1 \quad (2)$$

$$(1) + (-1) \cdot (2): 6x = 8$$

$$x = \frac{8}{6} = \frac{4}{3}$$

$$x = \frac{4}{3} \text{ in (2) einsetzen: } -\frac{4}{3} + y = -1 = -\frac{3}{3}$$

$$y = \frac{1}{3}$$

$$L = \left\{ \left(\frac{4}{3}, \frac{1}{3} \right) \right\}$$

Aufgabe 3.2

$$x + y = 1 \quad (1)$$

$$6x + 5y = 5 \quad (2)$$

$$(-5) \cdot (1) + (2): x = 0$$

$$x = 0 \text{ in (1) einsetzen: } y = 1$$

$$L = \{(0, 1)\}$$

Aufgabe 3.3

$$-2x + y = 1 \quad (1)$$

$$9x - 4y = 8 \quad (2)$$

$$4 \cdot (1) + (2): x = 12$$

$$x = 12 \text{ in (1) einsetzen: } -24 + y = 1$$

$$y = 25$$

$$L = \{(12, 25)\}$$

Aufgabe 3.4

$$-4x + 3y = -2 \quad (1)$$

$$-5x + 4y = 2 \quad (2)$$

$$-4 \cdot (1) + 3 \cdot (2): 16x - 15x = 8 + 6$$

$$x = 14$$

$$x = 14 \text{ in (1) einsetzen: } -56 + 3y = -2$$

$$3y = 54$$

$$y = 18$$

$$L = \{(14, 18)\}$$

Aufgabe 3.5

$$x + 3y = 1 \quad (1)$$

$$8x - 9y = -3 \quad (2)$$

$$3 \cdot (1) + (2): 11x = 0 \quad \Rightarrow \quad x = 0$$

$$x = 0 \text{ in (1) einsetzen: } 0 + 3y = 1 \quad \Rightarrow \quad y = \frac{1}{3}$$

$$L = \{(0, \frac{1}{3})\}$$

Aufgabe 3.6

$$-7x + 4y = -9 \quad (1)$$

$$-8x + 7y = -3 \quad (2)$$

$$-7 \cdot (1) + 4 \cdot (2): 49x + (-32x) = 63 + (-12)$$

$$17x = 51$$

$$x = 3$$

$$x = 3 \text{ in (1) einsetzen: } -21 + 4y = -9$$

$$4y = 12$$

$$y = 3$$

$$L = \{(3, 3)\}$$

Aufgabe 3.7

$$-5x - y = 5 \quad (1)$$

$$-x - y = -3 \quad (2)$$

$$-1 \cdot (1) + (2): 4x = -8$$

$$x = -2$$

$$x = -2 \text{ in (2) einsetzen: } 2 - y = -3y = 5$$

$$L = \{(-2, 5)\}$$

Aufgabe 3.8

$$-7x - 5y = 1 \quad (1)$$

$$3x - y = -2 \quad (2)$$

$$(1) + (-5) \cdot (2): -22x = 11$$

$$x = -\frac{1}{2}$$

$$x = -\frac{1}{2} \text{ in (2) einsetzen: } -\frac{3}{2} - y = -2 = -\frac{4}{2}$$

$$y = \frac{1}{2}$$

$$L = \{(-\frac{1}{2}, \frac{1}{2})\}$$

Aufgabe 3.9

$$x + y = -9 \quad (1)$$

$$8x + 3y = 8 \quad (2)$$

$$-3 \cdot (1) + \cdot (2): 5x = 35$$

$$x = 7$$

$$x = 7 \text{ in (1) einsetzen: } 7 + y = -9$$

$$y = -16$$

$$L = \{(7, -16)\}$$

Aufgabe 3.10

$$4x - 2y = -9 \quad (1)$$

$$8x - y = 3 \quad (2)$$

$$(1) + (-2) \cdot (2): 4x - 16x = -9 - 6$$

$$-12x = -15$$

$$x = \frac{15}{12} = \frac{5}{4}$$

$$x = \frac{5}{4} \text{ in (2) einsetzen: } 8 \cdot \frac{5}{4} - y = 3$$

$$10 - y = 3$$

$$y = 7$$

$$L = \{(\frac{5}{4}, 7)\}$$

Aufgabe 4.1

$$x + y + z = 22$$

$$x - y + z = 16$$

$$x + y - z = 4$$

$$\begin{array}{cccc} 1 & 1 & 1 & 22 \\ 1 & -1 & 1 & 16 \\ 1 & 1 & -1 & 4 \end{array} \begin{array}{l} \left. \begin{array}{l} \cdot (-1) \\ \leftarrow + \end{array} \right\} \\ \left. \begin{array}{l} \cdot (-1) \\ \leftarrow + \end{array} \right\} \end{array}$$

$$\begin{array}{cccc} 1 & 1 & 1 & 22 \\ 0 & -2 & 0 & -6 \\ 0 & 0 & -2 & -18 \end{array} \begin{array}{l} \\ \cdot (-\frac{1}{2}) \\ \cdot (-\frac{1}{2}) \end{array}$$

$$\begin{array}{cccc} 1 & 1 & 1 & 22 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 9 \end{array}$$

$$x + y + z = 22$$

$$y = 3$$

$$z = 9$$

Rückwärtseinsetzen:

$$y = 3$$

$$x = 22 - y - z = 22 - 3 - 9 = 10$$

$$L = \{(10, 3, 9)\}$$

Aufgabe 4.2

$$x + y - z = 16$$

$$x - y + z = 20$$

$$x - y - z = 4$$

$$\begin{array}{cccc} 1 & 1 & -1 & 16 \\ 1 & -1 & 1 & 20 \\ 1 & -1 & -1 & 4 \end{array} \begin{array}{l} \left. \begin{array}{l} \cdot (-1) \\ \leftarrow + \end{array} \right\} \\ \left. \begin{array}{l} \cdot (-1) \\ \leftarrow + \end{array} \right\} \end{array}$$

$$\begin{array}{cccc} 1 & 1 & -1 & 16 \\ 0 & -2 & 2 & 4 \\ 0 & -2 & 0 & -12 \end{array} \cdot (-\frac{1}{2})$$

$$\begin{array}{cccc} 1 & 1 & -1 & 16 \\ 0 & 1 & -1 & -2 \\ 0 & -2 & 0 & -12 \end{array} \begin{array}{l} \left. \begin{array}{l} \cdot 2 \\ \leftarrow + \end{array} \right\} \end{array}$$

$$\begin{array}{cccc} 1 & 1 & -1 & 16 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & -2 & -16 \end{array} \cdot (-\frac{1}{2})$$

$$\begin{array}{cccc} 1 & 1 & -1 & 16 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 2 \end{array} \cdot (-\frac{1}{2})$$

$$x + y - z = 16$$

$$y - z = -2$$

$$z = 8$$

Rückwärtseinsetzen:

$$y = -2 + z = -2 + 8 = 6$$

$$x = 16 - y + z = 16 - 6 + 8 = 18$$

$$L = \{(18, 6, 8)\}$$

Aufgabe 4.3

$$\begin{aligned}x - y - z &= 5 \\ -2x + y + 7z &= -2 \\ 3x - z &= 8\end{aligned}$$

$$\begin{array}{cccc|c} 1 & -1 & -1 & 5 & \\ -2 & 1 & 7 & -2 & \\ 3 & 0 & -1 & 8 & \end{array} \begin{array}{l} \left. \begin{array}{l} \cdot 2 \\ \leftarrow + \end{array} \right\} \\ \left. \begin{array}{l} \cdot (-3) \\ \leftarrow + \end{array} \right\} \end{array}$$

$$\begin{array}{cccc|c} 1 & -1 & -1 & 5 & \\ 0 & -1 & 5 & 8 & \cdot (-1) \\ 0 & 3 & 2 & -7 & \end{array}$$

$$\begin{array}{cccc|c} 1 & -1 & -1 & 5 & \\ 0 & 1 & -5 & -8 & \\ 0 & 3 & 2 & -7 & \left. \begin{array}{l} \cdot (-3) \\ \leftarrow + \end{array} \right\} \end{array}$$

$$\begin{array}{cccc|c} 1 & -1 & -1 & 5 & \\ 0 & 1 & -5 & -8 & \\ 0 & 0 & 17 & 17 & \cdot \frac{1}{17} \end{array}$$

$$\begin{array}{cccc} 1 & -1 & -1 & 5 \\ 0 & 1 & -5 & -8 \\ 0 & 0 & 1 & 1 \end{array}$$

$$\begin{aligned}x - y - z &= 5 \\ y - 5z &= -8 \\ z &= 1\end{aligned}$$

Rückwärtseinsetzen:

$$y = -8 + 5z = -8 + 5 = -3$$

$$x = 5 + y + z = 5 - 3 + 1 = 3$$

$$L = \{(3, -3, 1)\}$$

Aufgabe 4.4

$$x + 2y - z = -1$$

$$2x - y + z = -1$$

$$3x + y + 2z = 2$$

$$\begin{array}{cccc} 1 & 2 & -1 & -1 \\ 2 & -1 & 1 & -1 \\ 3 & 1 & 2 & 2 \end{array} \begin{array}{l} \left. \begin{array}{l} \cdot (-2) \\ \leftarrow + \end{array} \right\} \\ \left. \begin{array}{l} \cdot (-3) \\ \leftarrow + \end{array} \right\} \end{array}$$

$$\begin{array}{cccc} 1 & 2 & -1 & -1 \\ 0 & -5 & 3 & 1 \\ 0 & -5 & 5 & 5 \end{array} \cdot (-\frac{1}{5})$$

$$\begin{array}{cccc} 1 & 2 & -1 & -1 \\ 0 & -5 & 3 & 1 \\ 0 & 1 & -1 & -1 \end{array} \left. \begin{array}{l} \leftarrow \\ \leftarrow \end{array} \right\}$$

$$\begin{array}{cccc} 1 & 2 & -1 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & -5 & 3 & 1 \end{array} \left. \begin{array}{l} \leftarrow \cdot 5 \\ \leftarrow + \end{array} \right\}$$

$$\begin{array}{cccc} 1 & 2 & -1 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -2 & -4 \end{array} \cdot (-\frac{1}{2})$$

$$\begin{array}{cccc} 1 & 2 & -1 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{array}$$

$$x + 2y - z = -1$$

$$y - z = -1$$

$$z = 2$$

Rückwärtseinsetzen:

$$y = -1 + z = -1 + 2 = 1$$

$$x = -1 - 2y + z = -1 - 2 + 2 = -1$$

$$L = \{(-1, 1, 2)\}$$

Aufgabe 4.5

$$4x - y - z = -1$$

$$3x - 2y + 4z = 3$$

$$x - y + 3z = 4$$

$$\begin{array}{cccc} 4 & -1 & -1 & -1 \\ 3 & -2 & 4 & 3 \\ 1 & -1 & 3 & 4 \end{array} \begin{array}{l} \left. \begin{array}{l} \leftarrow \\ \leftarrow \\ \leftarrow \end{array} \right\} \end{array}$$

$$\begin{array}{cccc} 1 & -1 & 3 & 4 \\ 3 & -2 & 4 & 3 \\ 4 & -1 & -1 & -1 \end{array} \begin{array}{l} \left. \begin{array}{l} \leftarrow + \\ \leftarrow + \end{array} \right\} \cdot (-3) \\ \left. \begin{array}{l} \leftarrow + \\ \leftarrow + \end{array} \right\} \cdot (-4) \end{array}$$

$$\begin{array}{cccc} 1 & -1 & 3 & 4 \\ 0 & 1 & -5 & -9 \\ 0 & 3 & -13 & -17 \end{array} \begin{array}{l} \left. \begin{array}{l} \leftarrow + \\ \leftarrow + \end{array} \right\} \cdot (-3) \end{array}$$

$$\begin{array}{cccc} 1 & -1 & 3 & 4 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 2 & 10 \end{array} \cdot \frac{1}{2}$$

$$\begin{array}{cccc} 1 & -1 & 3 & 4 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 5 \end{array}$$

$$x - y + 3z = 4$$

$$y - 5z = -9$$

$$z = 5$$

Rückwärtseinsetzen:

$$y = -9 + 5z = -9 + 25 = 16$$

$$x = 4 + y - 3z = 4 + 16 - 15 = 5$$

$$L = \{(5, 16, 5)\}$$

Aufgabe 4.6

$$\begin{aligned} 3x_1 + 5x_2 - 3x_3 - 2x_4 &= 5 \\ 2x_1 - 3x_2 - 4x_3 - 2x_4 &= 4 \\ -3x_1 - 3x_2 + 4x_3 + x_4 &= -2 \\ x_1 - x_2 - 2x_3 - x_4 &= 0 \end{aligned}$$

$$\begin{array}{ccccc} 3 & 5 & -3 & -2 & 5 \\ 2 & -3 & -4 & -2 & 4 \\ -3 & -3 & 4 & 1 & -2 \\ 1 & -1 & -2 & -1 & 0 \end{array} \begin{array}{l} \left. \begin{array}{l} \leftarrow \\ \leftarrow \\ \leftarrow \\ \leftarrow \end{array} \right\} \end{array}$$

$$\begin{array}{ccccc} 1 & -1 & -2 & -1 & 0 \\ 2 & -3 & -4 & -2 & 4 \\ -3 & -3 & 4 & 1 & -2 \\ 3 & 5 & -3 & -2 & 5 \end{array} \begin{array}{l} \left. \begin{array}{l} \leftarrow \cdot (-2) \\ \leftarrow + \end{array} \right\} \\ \left. \begin{array}{l} \leftarrow \cdot 3 \\ \leftarrow + \end{array} \right\} \\ \left. \begin{array}{l} \leftarrow \cdot (-3) \\ \leftarrow + \end{array} \right\} \end{array}$$

$$\begin{array}{ccccc} 1 & -1 & -2 & -1 & 0 \\ 0 & -1 & 0 & 0 & 4 \\ 0 & -6 & -2 & -2 & -2 \\ 0 & 8 & 3 & 1 & 5 \end{array} \begin{array}{l} \\ \cdot (-1) \\ \\ \end{array}$$

$$\begin{array}{ccccc} 1 & -1 & -2 & -1 & 0 \\ 0 & 1 & 0 & 0 & -4 \\ 0 & -6 & -2 & -2 & -2 \\ 0 & 8 & 3 & 1 & 5 \end{array} \begin{array}{l} \\ \left. \begin{array}{l} \leftarrow \cdot 6 \\ \leftarrow + \end{array} \right\} \\ \left. \begin{array}{l} \leftarrow \cdot (-8) \\ \leftarrow + \end{array} \right\} \end{array}$$

$$\begin{array}{ccccc} 1 & -1 & -2 & -1 & 0 \\ 0 & 1 & 0 & 0 & -4 \\ 0 & 0 & -2 & -2 & -26 \\ 0 & 0 & 3 & 1 & 37 \end{array} \begin{array}{l} \\ \\ \cdot (-\frac{1}{2}) \\ \end{array}$$

$$\begin{array}{ccccc} 1 & -1 & -2 & -1 & 0 \\ 0 & 1 & 0 & 0 & -4 \\ 0 & 0 & 1 & 1 & 13 \\ 0 & 0 & 3 & 1 & 37 \end{array} \begin{array}{l} \\ \\ \left. \begin{array}{l} \leftarrow \cdot (-3) \\ \leftarrow + \end{array} \right\} \\ \end{array}$$

$$\begin{array}{ccccc} 1 & -1 & -2 & -1 & 0 \\ 0 & 1 & 0 & 0 & -4 \\ 0 & 0 & 1 & 1 & 26 \\ 0 & 0 & 0 & -2 & -2 \end{array} \begin{array}{l} \\ \\ \\ \cdot (-\frac{1}{2}) \end{array}$$

$$\begin{array}{ccccc} 1 & -1 & -2 & -1 & 0 \\ 0 & 1 & 0 & 0 & -4 \\ 0 & 0 & 1 & 1 & 26 \\ 0 & 0 & 0 & 1 & 1 \end{array}$$

$$\begin{aligned} x_1 - x_2 - 2x_3 - x_4 &= 0 \\ x_2 &= -4 \\ x_3 + x_4 &= 13 \\ x_4 &= 1 \end{aligned}$$

Rückwärtseinsetzen:

$$x_3 = 13 - x_4 = 13 - 1 = 12$$

$$x_2 = -4$$

$$x_1 = x_2 + 2x_3 + x_4 = -4 + 24 + 1 = 21$$

$$L = \{(21, -4, 12, 1)\}$$

Aufgabe 5.1

Substitution: $x^2 = a, y^2 = b$

$$2x^2 + y^2 = 43 \quad \Rightarrow \quad 2a + b = 43 \quad (1)$$

$$6x^2 - y^2 = 29 \quad \Rightarrow \quad 6a - b = 29 \quad (2)$$

$$(1)+(2): 8a = 72 \quad \Rightarrow \quad a = 9$$

$$a = 9 \text{ in (1) einsetzen: } 18 + b = 43 \quad \Rightarrow \quad b = 25$$

$$\text{Resubstitution: } a = 9 = x^2 \quad \Rightarrow \quad x_{1,2} = \pm 3$$

$$b = 25 = y^2 \quad \Rightarrow \quad y_{1,2} = \pm 5$$

$$L = \{(3, 5), (-3, 5), (3, -5), (-3, -5)\}$$

Aufgabe 5.2

Substitution: $x^2 = a, y^2 = b$

$$2x^2 + 5y^2 = -3 \quad \Rightarrow \quad 2a + 5b = -3 \quad (1)$$

$$7x^2 - 6y^2 = 13 \quad \Rightarrow \quad 7a - 6b = 13 \quad (2)$$

$$6 \cdot (1) + 5 \cdot (2): 12a + 35a = -18 + 65$$

$$47a = 47$$

$$a = 1$$

$$a = 1 \text{ in (1) einsetzen: } 2 + 5b = -3 \quad \Rightarrow \quad b = -1$$

$$\text{Resubstitution: } a = 1 = x^2 \quad \Rightarrow \quad x_{1,2} = \pm 1$$

$$b = -1 = y^2 \quad \Rightarrow \quad \text{keine Lösung}$$

$$L = \{ \}$$

Aufgabe 5.3

Substitution: $x^2 = a, y^2 = b$

$$4x^2 - y^2 = 8 \quad \Rightarrow \quad 4a - b = 8 \quad (1)$$

$$-3x^2 + 4y^2 = -6 \quad \Rightarrow \quad -3a + 4b = -6 \quad (2)$$

$$4 \cdot (1) + (2): 16a - 3a = 32 - 6$$

$$13a = 26$$

$$a = 2$$

$$a = 2 \text{ in (1) einsetzen: } 8 - b = 8 \quad \Rightarrow \quad b = 0$$

$$\text{Resubstitution: } a = 2 = x^2 \quad \Rightarrow \quad x_{1,2} = \pm\sqrt{2}$$

$$b = 0 = y^2 \quad \Rightarrow \quad y_1 = 0$$

$$L = \{(\sqrt{2}, 0), (-\sqrt{2}, 0)\}$$

Aufgabe 5.4

Substitution: $\sqrt{x} = a$, $\sqrt{y} = b$

$$-3\sqrt{x} + \sqrt{y} = -11 \quad \Rightarrow \quad -3a + b = -11 \quad (1)$$

$$4\sqrt{x} - \sqrt{y} = 19 \quad \Rightarrow \quad 4a - b = 19 \quad (2)$$

$$(1) + (2): a = 8$$

$$a = 8 \text{ in (1) einsetzen: } -24 + b = -11 \quad \Rightarrow \quad b = 13$$

$$\text{Resubstitution: } a = 8 = \sqrt{x} \quad \Rightarrow \quad x = 64$$

$$b = 13 = \sqrt{y} \quad \Rightarrow \quad y = 169$$

$$L = \{(64, 169)\}$$

Aufgabe 5.5

Substitution: $\sqrt{x} = a$, $\sqrt{y} = b$

$$-7\sqrt{x} + 6\sqrt{y} = 1 \quad \Rightarrow \quad -7a + 6b = 1 \quad (1)$$

$$-5\sqrt{x} + \sqrt{y} = -19 \quad \Rightarrow \quad -5a + b = -19 \quad (2)$$

$$(1) + (-6) \cdot (2): -7a + 30a = 1 + 114$$

$$23a = 115$$

$$a = 5$$

$$a = 5 \text{ in (2) einsetzen: } -25 + b = -19 \quad \Rightarrow \quad b = 6$$

$$\text{Resubstitution: } a = 5 = \sqrt{x} \quad \Rightarrow \quad x = 25$$

$$b = 6 = \sqrt{y} \quad \Rightarrow \quad y = 36$$

$$L = \{(25, 36)\}$$

Aufgabe 5.6

Substitution: $\sqrt{x} = a$, $\sqrt{y} = b$

$$7\sqrt{x} + \sqrt{y} = -5 \quad \Rightarrow \quad 7a + b = -5 \quad (1)$$

$$\sqrt{x} + 8\sqrt{y} = 15 \quad \Rightarrow \quad a + 8b = 15 \quad (2)$$

$$-8 \cdot (1) + (2): -56a + a = 40 + 15$$

$$-55a = 55$$

$$a = -1$$

$$a = -1 \text{ in (1) einsetzen: } -7 + b = -5 \quad \Rightarrow \quad b = 2$$

$$\text{Resubstitution: } a = -1 = \sqrt{x} \quad \Rightarrow \quad \text{keine Lösung}$$

$$b = 2 = \sqrt{y} \quad \Rightarrow \quad y = 4$$

$$L = \{\}$$

Aufgabe 5.7

Substitution: $\frac{1}{x} = a, \frac{1}{y} = b$

$$-\frac{1}{x} - \frac{1}{y} = 1 \quad \Rightarrow \quad -a - b = 1 \quad (1)$$

$$\frac{4}{x} + \frac{1}{y} = 5 \quad \Rightarrow \quad 4a + b = 5 \quad (2)$$

$$(1) + (2): 3a = 6 \quad \Rightarrow \quad a = 2$$

$$a = 2 \text{ in } (2) \text{ einsetzen: } 8 + b = 5 \quad \Rightarrow \quad b = -3$$

$$\begin{aligned} \text{Resubstitution: } a = 2 = \frac{1}{x} &\Rightarrow x = \frac{1}{2} \\ b = -3 = \frac{1}{y} &\Rightarrow y = -\frac{1}{3} \end{aligned}$$

$$L = \left\{ \left(\frac{1}{2}, -\frac{1}{3} \right) \right\}$$

Aufgabe 5.8

Substitution: $\frac{1}{x} = a, \frac{1}{y} = b$

$$\frac{4}{x} + \frac{7}{y} = 28 \quad \Rightarrow \quad 4a + 7b = 28 \quad (1)$$

$$-\frac{1}{x} + \frac{4}{y} = 16 \quad \Rightarrow \quad -a + 4b = 16 \quad (2)$$

$$(1) + 4 \cdot (2): 7b + 16b = 28 + 64$$

$$23b = 92$$

$$b = 4$$

$$b = 4 \text{ in } (2) \text{ einsetzen: } -a + 16 = 16 \quad \Rightarrow \quad a = 0$$

$$\begin{aligned} \text{Resubstitution: } a = 0 = \frac{1}{x} &\Rightarrow \text{keine Lösung} \\ b = 4 = \frac{1}{y} &\Rightarrow y = \frac{1}{4} \end{aligned}$$

$$L = \{ \}$$

Aufgabe 5.9

Substitution: $\frac{1}{x} = a, \frac{1}{y} = b$

$$\frac{2}{x} - \frac{1}{y} = \frac{7}{2} \quad \Rightarrow \quad 2a - b = \frac{7}{2} \quad (1)$$

$$-\frac{1}{x} - \frac{2}{y} = \frac{13}{4} \quad \Rightarrow \quad -a - 2b = \frac{13}{4} \quad (2)$$

$$(1) + 2 \cdot (2): -5b = \frac{7}{2} + \frac{13}{2} = 10 \quad \Rightarrow \quad b = -2$$

$$\begin{aligned} b = -2 \text{ in } (2) \text{ einsetzen: } -a + 4 &= \frac{13}{4} \\ a &= 4 - \frac{13}{4} = \frac{3}{4} \end{aligned}$$

$$\begin{aligned} \text{Resubstitution: } a = \frac{3}{4} = \frac{1}{x} &\Rightarrow x = \frac{4}{3} \\ b = -2 = \frac{1}{y} &\Rightarrow y = -\frac{1}{2} \end{aligned}$$

$$L = \left\{ \left(\frac{4}{3}, -\frac{1}{2} \right) \right\}$$