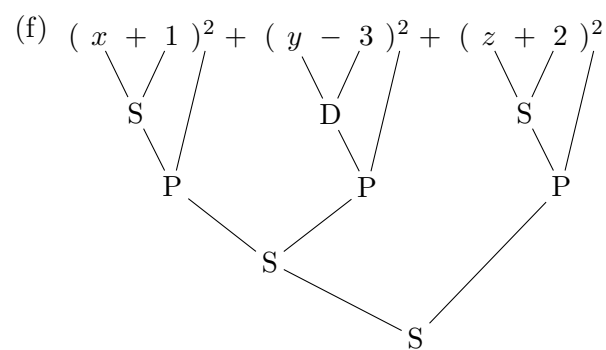
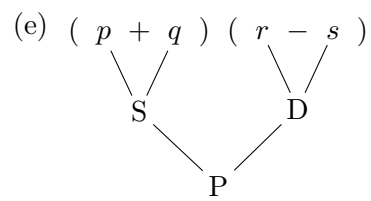
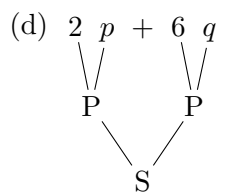
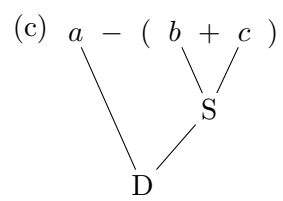
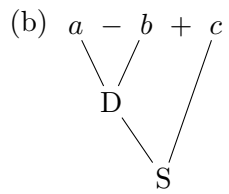
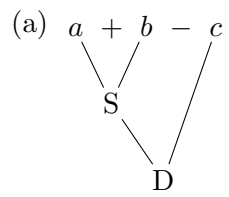


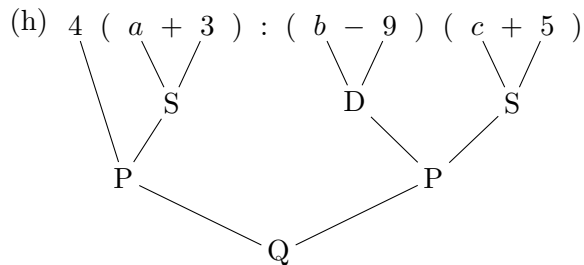
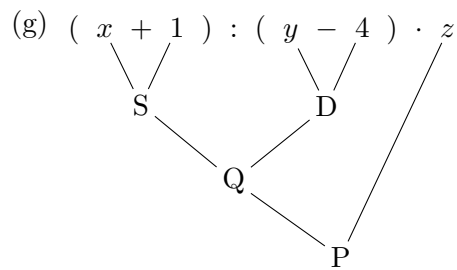
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**Algebra mit Bruchtermen**  
**Lösungen+**

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### Aufgabe 1.1





### Aufgabe 1.2

$$cd + ce = c(d + e)$$

### Aufgabe 1.3

$$ct - dt^2 = t(c - dt)$$

### Aufgabe 1.4

$$81y^3 + 54y = 27y(3y^2 + 2)$$

### Aufgabe 1.5

$$10at + 15bt - 6ct = t(10a + 15b - 6c)$$

### Aufgabe 1.6

$$4x^2yz - 10xy^2z + 16xyz^2 = 2xyz(2x - 5y + 8z)$$

### Aufgabe 1.7

$$(2n - 2)(3n - 3) = 2(n - 1) \cdot 3(n - 1) = 6(n - 1)^2$$

### Aufgabe 1.8

$$\begin{aligned} r(2u + 3v) - r(u + v) &= r((2u + 3v) - (u + v)) \\ &= r(2u + 3v - u - v) = r(u + 2v) \end{aligned}$$

### Aufgabe 1.9

$$\begin{aligned} 3p^2(u - v) - 2p(u - v) - 8(u - v) + (u - v) \\ = (u - v)(3p^2 - 2p - 8 + 1) = (u - v)(3p^2 - 2p - 7) \end{aligned}$$

**Aufgabe 1.10**

$$\begin{aligned}(3u + v)(u - v) - (3u + v)(u - w) &= (3u + v)((u - v) - (u - w)) \\ &= (3u + v)(u - v - u + w) \\ &= (3u + v)(w - v)\end{aligned}$$

**Aufgabe 1.11**

$$\begin{aligned}a(x + y) + 2x + 2y &= a(x + y) + 2(x + y) \\ &= (x + y)(a + 2)\end{aligned}$$

**Aufgabe 1.12**

$$4m(p + q) - p - q = 4m(p + q) - 1(p + q) = (p + q)(4m - 1)$$

**Aufgabe 1.13**

$$\begin{aligned}mn - m + n - 1 &= m(n - 1) + 1(n - 1) \\ &= (n - 1)(m + 1)\end{aligned}$$

**Aufgabe 1.14**

$$\begin{aligned}as + at + bs + bt + cs + ct &= a(s + t) + b(s + t) + c(s + t) \\ &= (s + t)(a + b + c)\end{aligned}$$

**Aufgabe 1.15**

$$z^2 - 225 = (z + 15)(z - 15)$$

**Aufgabe 1.16**

$$\begin{aligned}u^2v^2 - 64w^2 &= (uv)^2 - 64w^2 \\ &= (uv - 8w)(uv + 8w)\end{aligned}$$

**Aufgabe 1.17**

$$4c^2 + 28cd + 49d^2 = (2c + 7d)^2$$

**Aufgabe 1.18**

$$\begin{aligned}p^2 - x^2 - 2x - 1 &= p^2 - (x^2 + 2x + 1) \\ &= p^2 - (x + 1)^2 \\ &= (p + (x + 1))(p - (x + 1)) \\ &= (p + x + 1)(p - x - 1)\end{aligned}$$

**Aufgabe 1.19**

$$z^2 - 19z + 48 = (z - 3)(z - 16)$$

**Aufgabe 1.20**

$$5k^2 - 2k - 3 = (5k + 3)(k - 1)$$

**Aufgabe 1.21**

$$m^4 - 5m^2n - 24n^2 = (m^2 + 3n)(m^2 - 8n)$$

### Aufgabe 2.1

$$\begin{array}{r} (y^3 - 10y^2 + 16y + 48) : (y - 6) = y^2 - 4y - 8 \\ - (y^3 - 6y^2) \\ \hline -4y^2 + 16y \\ - (-4y^2 + 24y) \\ \hline -8y + 48 \\ - (-8y + 48) \\ \hline 0 \end{array}$$

### Aufgabe 2.2

$$\begin{array}{r} (c^3 + 1.5c^2 - 2c - 20) : (2c - 5) = 0.5c^2 + 2c + 4 \\ - (c^3 - 2.5c^2) \\ \hline 4c^2 - 2c \\ - (4c^2 - 10c) \\ \hline 8c - 20 \\ - (8c - 20) \\ \hline 0 \end{array}$$

### Aufgabe 2.3

$$\begin{array}{r} (z^3 + 9z^2 + 0z - 100) : (z + 5) = z^2 + 4z - 20 \\ - (z^3 + 5z^2) \\ \hline 4z^2 + 0z \\ - (4z^2 + 20z) \\ \hline -20z - 100 \\ - (-20z - 100) \\ \hline 0 \end{array}$$

### Aufgabe 2.4

$$\begin{array}{r} (n^4 + 0n^3 + 0n^2 + 5n - 6) : (n + 2) = n^3 - 2n^2 + 4n - 3 \\ - (n^4 + 2n^3) \\ \hline -2n^3 + 0n^2 \\ - (-2n^3 - 4n^2) \\ \hline 4n^2 + 5n \\ - (4n^2 + 8n) \\ \hline -3n - 6 \\ - (-3n - 6) \\ \hline 0 \end{array}$$

### Aufgabe 2.5

$$\begin{aligned} & (4r^3 + \frac{2}{3}r^2 + \frac{5}{3}r + 2) : (3r + 2) = \frac{4}{3}r^2 - \frac{2}{3}r + 1 \\ & - (4r^3 + \frac{8}{3}r^2) \\ & \quad \underline{-\frac{6}{3}r^2 + \frac{5}{3}r} \\ & - (-\frac{6}{3}r^2 - \frac{4}{3}r) \\ & \quad \underline{3r + 2} \\ & \quad \quad \underline{- (3r + 2)} \\ & \quad \quad \quad 0 \end{aligned}$$

### Aufgabe 2.6

$$\begin{aligned} & (k^5 + 0k^4 + 0k^3 + 0k^2 + 0k - 1) : (k - 1) = k^4 + k^3 + k^2 + k + 1 \\ & - (k^5 - k^4) \\ & \quad \underline{k^4 + 0k^3} \\ & - (k^4 - k^3) \\ & \quad \underline{k^3 + 0k^2} \\ & - (k^3 - k^2) \\ & \quad \underline{k^2 + 0k} \\ & - (k^2 - k) \\ & \quad \underline{k - 1} \\ & - (k - 1) \\ & \quad \underline{0} \end{aligned}$$

### Aufgabe 2.7

$$\begin{aligned} & (x^4 - 8x^3 + 14x^2 - 7x - 30) : (x^2 - 3x + 5) = x^2 - 5x - 6 \\ & - (x^4 - 3x^3 + 5x^2) \\ & \quad \underline{-5x^3 + 9x^2 - 7x} \\ & - (-5x^3 + 15x^2 - 25x) \\ & \quad \underline{-6x^2 + 18x - 30} \\ & - (-6x^2 + 18x - 30) \\ & \quad \underline{0} \end{aligned}$$

### Aufgabe 2.8

$$\begin{aligned} & (9p^4 + 0p^3 - 31p^2 + 0p + 25) : (3p^2 + p - 5) = 3p^2 - p - 5 \\ & - (9p^4 + 3p^3 - 15p^2) \\ & \quad \underline{-3p^3 - 16p^2 + 0p} \\ & - (-3p^3 - p^2 + 5p) \\ & \quad \underline{-15p^2 - 5p + 25} \\ & - (-15p^2 - 5p + 25) \\ & \quad \underline{0} \end{aligned}$$

### Aufgabe 2.9

$$\begin{aligned} & (u^5 - 3u^4 + 0u^3 - 9u^2 - 16u + 12) : (u^2 + 4) = u^3 - 3u^2 - 4u + 3 \\ & - \frac{(u^5 + 4u^3)}{-3u^4 - 4u^3 - 9u^2} \\ & - \frac{(-3u^4 - 12u^2)}{-4u^3 + 3u^2 - 16u} \\ & - \frac{(-4u^3 - 16u)}{3u^2 + 12} \\ & - \frac{(3u^2 + 12)}{0} \end{aligned}$$

### Aufgabe 2.10

$$\begin{aligned} & (6z^4 + 8z^3 - 19z^2 - 7z - 12) : (3z^2 - 2z - 4) = 2z^2 + 4z - 1 + \frac{7z - 16}{3z^2 - 2z - 4} \\ & - \frac{(6z^4 - 4z^3 - 8z^2)}{12z^3 - 11z^2 - 7z} \\ & - \frac{(12z^3 - 8z^2 - 16z)}{-3z^2 + 9z - 12} \\ & - \frac{(-3z^2 + 2z + 4)}{7z - 16} \end{aligned}$$



### Aufgabe 3.1

$$\begin{array}{r} -9xy = -3 \cdot 3 \cdot x \cdot y \\ 45xz = 3 \cdot 3 \cdot 5 \cdot x \cdot z \\ \hline \text{ggT} : 3 \cdot 3 \cdot x = 9x \\ \text{kgV} : 3 \cdot 3 \cdot 5 \cdot x \cdot y \cdot z = 45xyz \end{array}$$

### Aufgabe 3.2

$$\begin{array}{r} uv = u \cdot v \\ vw = v \cdot w \\ \hline \text{ggT} : v = v \\ \text{kgV} : u \cdot v \cdot w = uvw \end{array}$$

### Aufgabe 3.3

$$\begin{array}{r} 6n^6 = 2 \cdot 3 \cdot n \cdot n \cdot n \cdot n \cdot n \cdot n \\ 9n^9 = 3 \cdot 3 \cdot n \cdot n \cdot n \cdot n \cdot n \cdot n \cdot n \\ \hline \text{ggT} : 3 \cdot n \cdot n \cdot n \cdot n \cdot n = 3n^6 \\ \text{kgV} : 2 \cdot 3 \cdot 3 \cdot n \cdot n \cdot n \cdot n \cdot n \cdot n \cdot n = 18n^9 \end{array}$$

### Aufgabe 3.4

$$\begin{array}{r} a = a \cdot 1 \\ a + b = (a + b) \cdot 1 \\ \hline \text{ggT} : 1 \\ \text{kgV} : a \cdot (a + b) \end{array}$$

### Aufgabe 3.5

$$\begin{array}{r} 4i + 4 = 4 \cdot (i + 1) \\ 5i + 5 = (i + 1) \cdot 5 \\ \hline \text{ggT} : (i + 1) = (i + 1) \\ \text{kgV} : 4 \cdot (i + 1) \cdot 5 = 20(i + 1) \end{array}$$

### Aufgabe 3.6

$$\begin{array}{r} z = z \\ z^2 - 3z = z \cdot (z - 3) \\ \hline \text{ggT} : z = z \\ \text{kgV} : z \cdot (z - 3) = z(z - 3) \end{array}$$

### Aufgabe 3.7

$$\begin{array}{r} 15t - 25 = 5 \cdot (3t - 5) \\ -6t + 10 = (3t - 5) \cdot (-2) \\ \hline \text{ggT} : (3t - 5) \\ \text{kgV} : 10 \cdot (3t - 5) \end{array}$$

### Aufgabe 3.8

$$\begin{array}{l} h^2 - h = h(h-1) \\ h - h^2 = -h(h-1) \\ \hline \text{ggT} : h(h-1) = h(h-1) \\ \text{kgV} : h(h-1) = h(h-1) \end{array}$$

### Aufgabe 3.9

$$\begin{array}{l} abc + b^2c = b c (a + b) \\ a^2b + ab^2 = a b (a + b) \\ \hline \text{ggT} : b c (a + b) = b c (a + b) \\ \text{kgV} : a b c (a + b) = abc(a + b) \end{array}$$

### Aufgabe 3.10

$$\begin{array}{l} 6p - 9 = 3(2p - 3) \\ 4p^2 - 6p = 2p(2p - 3) \\ \hline \text{ggT} : (2p - 3) = 2p - 3 \\ \text{kgV} : 2 \cdot 3 \cdot p(2p - 3) = 6p(2p - 3) \end{array}$$

### Aufgabe 3.11

$$\begin{array}{l} 8uv = 2 \cdot 2 \cdot 2 u v \\ 8u - 20v = 2 \cdot 2 (2u - 5v) \\ \hline \text{ggT} : 2 \cdot 2 = 4 \\ \text{kgV} : 2 \cdot 2 \cdot 2 u v \cdot 2u - 5v = 8uv(2u - 5v) \end{array}$$

### Aufgabe 3.12

$$\begin{array}{l} r^2 - 8r + 15 = (r - 3)(r - 5) \\ -(r^2 + r - 12) = -(r - 3)(r + 4) \\ \hline \text{ggT} : (r - 3) \\ \text{kgV} : (r - 3)(r - 5)(r + 4) \end{array}$$

### Aufgabe 3.13

$$\begin{aligned} x(x+y) + x^2 - y^2 &= x(x+y) + (x+y)(x-y) \\ &= (x+y)(x+(x-y)) \stackrel{*}{=} (x+y)(2x-y) \end{aligned}$$

$$\begin{array}{l} x(x+y) + x^2 - y^2 \stackrel{*}{=} (x+y)(2x-y) \\ 4x^2 - 4xy + y^2 = (2x-y)(2x-y) \\ \hline \text{ggT} : (2x-y) \\ \text{kgV} : (x+y)(2x-y)^2 \end{array}$$

### Aufgabe 3.14

$$\begin{array}{l} z^2 - 8z + 16 = (z - 4)(z - 4) \\ wz - 4w - 3z + 12 = (z - 4)(w - 3) \\ \hline \text{ggT} : (z - 4) \\ \text{kgV} : (z - 4)(z - 4)(w - 3) = (z - 4)^2(w - 3) \end{array}$$

### Aufgabe 3.15

$$\begin{array}{l} (2c - 18)^2 = 2 \cdot 2 \cdot (c - 9)(c - 9) \\ c^2 - 7c - 18 = (c - 9)(c + 2) \\ \hline \text{ggT} : (c - 9) = (c - 9) \\ \text{kgV} : 2 \cdot 2 \cdot (c - 9)(c - 9)(c + 2) = 4(c - 9)^2(c + 2) \end{array}$$

### Aufgabe 3.16

$$\begin{array}{l} 6a^2b = 2 \cdot 3 a a b \\ 15a^3b^2 = 3 a a b a b 5 \\ 18a^4b^4 = 2 \cdot 3 a a b a b a b b 3 \\ \hline \text{ggT} : 3 a a b = 3a^2b \\ \text{kgV} : 2 \cdot 3 a a b a b 5 a b b 3 = 90a^4b^4 \end{array}$$

### Aufgabe 3.17

$$\begin{array}{l} 2t - 5 = (2t - 5) \\ 10 - 4t = -(2t - 5) \cdot 2 \\ 6t - 15 = (2t - 5) \cdot 3 \\ \hline \text{ggT} : (2t - 5) \\ \text{kgV} : (2t - 5) \cdot 2 \cdot 3 = 6(2t - 5) \end{array}$$

### Aufgabe 3.18

$$\begin{array}{l} x^2 - 4y^2 = (x - 2y)(x + 2y) \\ x^2 - 4xy + 4y^2 = (x - 2y)(x - 2y) \\ x^2 - 2xy = (x - 2y)x \\ \hline \text{ggT} : (x - 2y) = x - 2y \\ \text{kgV} : (x - 2y)(x + 2y)(x - 2y)x = x(x + 2y)(x - 2y)^2 \end{array}$$

**Aufgabe 4.1**

$$\frac{10r}{15r} = \frac{2}{3}$$

**Aufgabe 4.2**

$$\frac{16xyz}{20xz} = \frac{4y}{5}$$

**Aufgabe 4.3**

$$\frac{25pq}{5q} = 5p$$

**Aufgabe 4.4**

$$\frac{-27s^2}{-36st} = \frac{3s}{4t}$$

**Aufgabe 4.5**

$$\frac{14n - 10}{7} = \frac{2(7n - 5)}{7}$$

**Aufgabe 4.6**

$$\frac{28x - 35y}{21} = \frac{7(4x - 5y)}{21} = \frac{4x - 5y}{3}$$

**Aufgabe 4.7**

$$\frac{92p + 46q}{23} = \frac{46(2p + q)}{23} = 2(2p + q) = 4p + 2q$$

**Aufgabe 4.8**

$$\frac{360z^2 - 90z}{45z^2} = \frac{90z(4z - 1)}{45z^2} = \frac{2(4z - 1)}{z} = \frac{8z - 2}{z}$$

**Aufgabe 4.9**

$$\frac{uw}{uv + uw} = \frac{uw}{u(v + w)} = \frac{w}{v + w}$$

**Aufgabe 4.10**

$$\frac{2m}{4mn - 2m} = \frac{2m}{2m(2n - 1)} = \frac{1}{2n - 1}$$

**Aufgabe 4.11**

$$\frac{-h}{h^2 + h} = \frac{-h}{h(h + 1)} = \frac{-1}{h + 1}$$

**Aufgabe 4.12**

$$\frac{18a^2bc}{18a^2b^2c + 54a^2bc^2} = \frac{18a^2bc}{18a^2bc(b + 3c)} = \frac{1}{b + 3c}$$

**Aufgabe 4.13**

$$\frac{2y + 2}{5 + 5y} = \frac{2(y + 1)}{5(1 + y)} = \frac{2(y + 1)}{5(y + 1)} = \frac{2}{5}$$

**Aufgabe 4.14**

$$\frac{rs - rt}{su - tu} = \frac{r(s - t)}{u(s - t)} = \frac{r}{u}$$

**Aufgabe 4.15**

$$\frac{a^2 - b^2}{3a + 3b} = \frac{(a - b)(a + b)}{3(a + b)} = \frac{a - b}{3}$$

**Aufgabe 4.16**

$$\frac{6u - 8v}{9u^2 - 16v^2} = \frac{2(3u - 4v)}{(3u - 4v)(3u + 4v)} = \frac{2}{3u + 4v}$$

**Aufgabe 4.17**

$$\frac{u^2 + 2uv + v^2}{4u + 4v} = \frac{(u + v)^2}{4(u + v)} = \frac{u + v}{4}$$

**Aufgabe 4.18**

$$\frac{2ac - 5bc}{4a^2 - 20ab + 25b^2} = \frac{c(2a - 5b)}{(2a - 5b)^2} = \frac{c}{2a - 5b}$$

**Aufgabe 4.19**

$$\frac{10m - 5}{8m^2 - 8m + 2} = \frac{5(2m - 1)}{2(4m^2 - 4m + 1)} = \frac{5(2m - 1)}{2(2m - 1)^2} = \frac{5}{2(2m - 1)}$$

**Aufgabe 4.20**

$$\begin{aligned} \frac{(16p - 16q)^2}{16p^2 - 16q^2} &= \frac{(16(p - q))^2}{16(p^2 - q^2)} = \frac{16^2(p - q)^2}{16(p - q)(p + q)} \\ &= \frac{16(p - q)}{p + q} \end{aligned}$$

**Aufgabe 4.21**

$$\begin{aligned}\frac{as + at + bs + bt}{2s + 2t} &= \frac{a(s + t) + b(s + t)}{2s + 2t} \\ &= \frac{(a + b)(s + t)}{2(s + t)} = \frac{a + b}{2}\end{aligned}$$

**Aufgabe 4.22**

$$\begin{aligned}\frac{375w - 1000}{6uw - 16u + 6vw - 16v} &= \frac{125(3w - 8)}{2u(3w - 8) + 2v(3w - 8)} \\ &= \frac{125(3w - 8)}{(3w - 8)(2u + 2v)} = \frac{125}{2u + 2v}\end{aligned}$$

**Aufgabe 4.23**

$$\begin{aligned}\frac{4af + 7ag - 8bf - 14bg}{3a - 6b} &= \frac{a(4f + 7g) - 2b(4f + 7g)}{3(a - 2b)} \\ &= \frac{(4f + 7g)(a - 2b)}{3(a - 2b)} = \frac{4f + 7g}{3}\end{aligned}$$

**Aufgabe 4.24**

$$\begin{aligned}\frac{kp - 5p + k - 5}{kp + k} &= \frac{p(k - 5) + (k - 5)}{k(p + 1)} \\ &= \frac{(k - 5)(p + 1)}{k(p + 1)} = \frac{k - 5}{k}\end{aligned}$$

**Aufgabe 4.25**

$$\frac{a^2 + 2a - 24}{a^2 - 6a + 8} = \frac{(a - 4)(a + 6)}{(a - 2)(a - 4)} = \frac{a + 6}{a - 2}$$

**Aufgabe 4.26**

$$\frac{kn - 2k}{3n^2 - 3n - 6} = \frac{k(n - 2)}{3(n^2 - n - 2)} = \frac{k(n - 2)}{3(n + 1)(n - 2)} = \frac{k}{3(n + 1)}$$

**Aufgabe 4.27**

$$\frac{r^2 - 8r + 7}{2r^2 - 4r + 2} = \frac{(r - 1)(r - 7)}{2(r^2 - 2r + 1)} = \frac{(r - 1)(r - 7)}{2(r - 1)^2} = \frac{r - 7}{2(r - 1)}$$

**Aufgabe 4.28**

$$\frac{a-b}{b-a} = \frac{(-1)(b-a)}{b-a} = -1$$

**Aufgabe 4.29**

$$\frac{4mp-20}{30-6mp} = \frac{-4(5-mp)}{6(5-mp)} = \frac{-4}{6} = -\frac{2}{3}$$

**Aufgabe 4.30**

$$\frac{k^2-13k+42}{14-2k} = \frac{(k-6)(k-7)}{-2(k-7)} = \frac{k-6}{-2} = \frac{-k+6}{2}$$

**Aufgabe 4.31**

$$\frac{3z-3y}{4y-4z} = \frac{3(z-y)}{4(y-z)} = \frac{-3(y-z)}{4(y-z)} = -\frac{3}{4}$$

**Aufgabe 4.32**

$$\frac{e^2-e}{1-e^2} = \frac{e(e-1)}{(1-e)(1+e)} = \frac{-e(1-e)}{(1-e)(1+e)} = \frac{-e}{1+e}$$

**Aufgabe 4.33**

$$\begin{aligned} \frac{c^2+c-20}{-c^2+c+30} &= \frac{(c-4)(c+5)}{-(c^2-c-30)} = \frac{(c-4)(c+5)}{-(c-6)(c+5)} \\ &= \frac{c-4}{-(c-6)} = \frac{c-4}{-c+6} \end{aligned}$$

**Aufgabe 4.34**

$$\frac{3n^2-9n}{n^3-2n^2-5n+6} \stackrel{*}{=} \frac{3n(n-3)}{(n-3)(n^2+n-2)} = \frac{3n}{n^2+n-2}$$

(\*) Falls ein Kürzen möglich ist, muss der Nenner den Faktor  $(n-3)$  enthalten. Dies lässt sich mit einer Polynomdivision nachweisen.

**Aufgabe 4.35**

$$\begin{aligned} \frac{4p^3-6p^2+2p+3}{4p^2-1} &= \frac{4p^3-6p^2+2p+3}{(2p-1)(2p+1)} \\ &= \frac{(2p^2-4p+3)(2p+1)}{(2p-1)(2p+1)} \\ &= \frac{2p^2-4p+3}{2p-1} \end{aligned}$$

**Aufgabe 4.36**

$$\frac{4y-x}{-3} = \frac{(-1)(4y-x)}{(-1)(-3)} = \frac{-4y+x}{3} = \frac{x-4y}{3}$$

**Aufgabe 4.37**

$$\frac{-uvw}{-u+v-w} = \frac{(-1)(-uvw)}{(-1)(-u+v-w)} = \frac{uvw}{u-v+w}$$

**Aufgabe 4.38**

$$\frac{1-qr}{1-q} = \frac{(-1)(1-qr)}{(-1)(1-q)} = \frac{qr-1}{q-1}$$

**Aufgabe 4.39**

$$\frac{-m-n}{n-m} = \frac{(-1)(-m-n)}{(-1)(n-m)} = \frac{m+n}{-n+m} = \frac{m+n}{m-n}$$

**Aufgabe 4.40**

$$\left[ \frac{7}{8w}, \frac{5}{6w} \right] = \left[ \frac{7 \cdot 3}{8w \cdot 3}, \frac{5 \cdot 4}{6w \cdot 4} \right] = \left[ \frac{21}{24w}, \frac{20}{24w} \right]$$

**Aufgabe 4.41**

$$\left[ \frac{p}{e^2}, \frac{p}{e^3} \right] = \left[ \frac{ep}{e^3}, \frac{p}{e^3} \right]$$

**Aufgabe 4.42**

$$\left[ \frac{x}{yz}, \frac{y}{xz}, \frac{z}{xy} \right] = \left[ \frac{x^2}{xyz}, \frac{y^2}{xyz}, \frac{z^2}{xyz} \right]$$

**Aufgabe 4.43**

$$\begin{aligned} \left[ \frac{15}{4mn^2}, \frac{25}{6m^3n} \right] &= \left[ \frac{15 \cdot 3m^2}{4mn^2 \cdot 3m^2}, \frac{25 \cdot 2n}{6m^3n \cdot 2n} \right] \\ &= \left[ \frac{45m^2}{12m^3n^2}, \frac{50n}{12m^3n^2} \right] \end{aligned}$$

**Aufgabe 4.44**

$$\left[ \frac{a}{b}, \frac{a}{b+c} \right] = \left[ \frac{a(b+c)}{b(b+c)}, \frac{ab}{b(b+c)} \right]$$

**Aufgabe 4.45**

$$\begin{aligned} \left[ \frac{q}{q^2-1}, \frac{q-1}{q+1} \right] &= \left[ \frac{q}{(q+1)(q-1)}, \frac{(q-1)(q-1)}{(q+1)(q-1)} \right] \\ &= \left[ \frac{q}{q^2-1}, \frac{(q-1)^2}{q^2-1} \right] \end{aligned}$$

**Aufgabe 4.46**

$$\left[ \frac{1}{t^2-t}, \frac{t-1}{t} \right] = \left[ \frac{1}{t(t-1)}, \frac{(t-1)^2}{t(t-1)} \right]$$



**Aufgabe 4.47**

$$\begin{aligned} \left[ \frac{3}{uv+v}, \frac{u+v}{2v} \right] &= \left[ \frac{3 \cdot 2}{v(u+1) \cdot 2}, \frac{(u+v) \cdot (u+1)}{2v \cdot (u+1)} \right] \\ &= \left[ \frac{6}{2v(u+1)}, \frac{(u+v)(u+1)}{2v(u+1)} \right] \end{aligned}$$

**Aufgabe 4.48**

$$\left[ \frac{w-z}{w+z}, \frac{w+z}{w-z} \right] = \left[ \frac{(w-z)^2}{w^2-z^2}, \frac{(w+z)^2}{w^2-z^2} \right]$$

**Aufgabe 4.49**

$$\begin{aligned} \left[ \frac{a}{a^2-b^2}, \frac{b}{b-a} \right] &= \left[ \frac{a}{(a-b)(a+b)}, \frac{-b}{a-b} \right] \\ &= \left[ \frac{a}{(a-b)(a+b)}, \frac{-b(a+b)}{(a-b)(a+b)} \right] \end{aligned}$$

**Aufgabe 4.50**

$$\begin{aligned} \left[ \frac{21}{2x-2}, \frac{-31}{3x-3}, \frac{41}{4x-4} \right] \\ = \left[ \frac{126}{12(x-1)}, \frac{-124}{12(x-1)}, \frac{123}{12(x-1)} \right] \end{aligned}$$

**Aufgabe 5.1**

$$\frac{7}{8a} - \frac{1}{8a} = \frac{7-1}{8a} = \frac{6}{8a} = \frac{3}{4a}$$

**Aufgabe 5.2**

$$\frac{5}{3n} + \frac{2}{3n} - \frac{-5}{3n} = \frac{5+2-(-5)}{3n} = \frac{12}{3n} = \frac{4}{n}$$

**Aufgabe 5.3**

$$\frac{5c}{12y} + \frac{c}{12y} = \frac{5c+c}{12y} = \frac{6c}{12y} = \frac{c}{2y}$$

**Aufgabe 5.4**

$$\frac{-76u}{35v} - \frac{8u}{35v} = \frac{-76u-8u}{35v} = \frac{-84u}{35v} = -\frac{12u}{5v}$$

**Aufgabe 5.5**

$$\begin{aligned} \frac{a+nb}{n} - \frac{a-nb}{n} &= \frac{a+nb-(a-nb)}{n} \\ &= \frac{a+nb-a+nb}{n} \\ &= \frac{2nb}{n} = 2b \end{aligned}$$

**Aufgabe 5.6**

$$\begin{aligned} -\frac{3r+4}{6} + \frac{5r+7}{6} &= \frac{-(3r+4)+(5r+7)}{6} \\ &= \frac{-3r-4+5r+7}{6} \\ &= \frac{2r+3}{6} \end{aligned}$$

**Aufgabe 5.7**

$$\begin{aligned} \frac{-t+7}{4t} - \frac{3t+4}{4t} - \frac{8t-5}{4t} &= \frac{-t+7-(3t+4)-(8t-5)}{4t} \\ &= \frac{-t+7-3t-4-8t+5}{4t} \\ &= \frac{-12t+8}{4t} \\ &= \frac{4(2-3t)}{4t} \\ &= \frac{2-3t}{t} \end{aligned}$$

**Aufgabe 5.8 (=5.7)**

$$\begin{aligned}
\frac{-t+7}{4t} - \frac{3t+4}{4t} - \frac{8t-5}{4t} &= \frac{-t+7 - (3t+4) - (8t-5)}{4t} \\
&= \frac{-t+7-3t-4-8t+5}{4t} \\
&= \frac{-12t+8}{4t} \\
&= \frac{4(2-3t)}{4t} \\
&= \frac{2-3t}{t}
\end{aligned}$$

**Aufgabe 5.9**

$$\frac{1}{m+1} + \frac{m}{m+1} = \frac{1+m}{m+1} = \frac{m+1}{m+1} = 1$$

**Aufgabe 5.10**

$$\frac{cd}{b-d} - \frac{bc}{b-d} = \frac{cd-bc}{b-d} = \frac{c(d-b)}{b-d} = \frac{-c(b-d)}{b-d} = -c$$

**Aufgabe 5.11**

$$\frac{q}{p-q} - \frac{p}{p-q} = \frac{q-p}{p-q} = \frac{(-1)(p-q)}{p-q} = \frac{-1}{1} = -1$$

**Aufgabe 5.12**

$$\frac{4ktw}{2t-1} - \frac{2kw}{2t-1} = \frac{4ktw-2kw}{2t-1} = \frac{2kw(2t-1)}{2t-1} = 2kw$$

**Aufgabe 5.13**

$$\frac{8}{9m} - \frac{11}{36m} = \frac{32}{36m} - \frac{11}{36m} = \frac{32-11}{36m} = \frac{21}{36m} = \frac{7}{12m}$$

**Aufgabe 5.14**

$$\frac{z}{n^2} + \frac{4}{3n} = \frac{3z}{3n^2} + \frac{4n}{3n^2} = \frac{3z+4n}{3n^2}$$

**Aufgabe 5.15**

$$\begin{aligned}
\frac{a+b}{b} - \frac{a-b}{a} &= \frac{(a+b)a}{ab} - \frac{b(a-b)}{ab} = \frac{a^2+ab - (ab-b^2)}{ab} \\
&= \frac{a^2+ab-ab+b^2}{ab} = \frac{a^2+b^2}{ab}
\end{aligned}$$

**Aufgabe 5.16**

$$\begin{aligned}\frac{x+y}{2xy} + \frac{x+z}{2xz} + \frac{y+z}{2yz} &= \frac{z(x+y)}{2xyz} + \frac{y(x+z)}{2xyz} + \frac{x(y+z)}{2xyz} \\ &= \frac{(xz+yz) + (xy+yz) + (xy+xz)}{2xyz} \\ &= \frac{2xy + 2yz + 2xz}{2xyz} \\ &= \frac{2(xy + yz + xz)}{2xyz} \\ &= \frac{xy + yz + xz}{xyz}\end{aligned}$$

**Aufgabe 5.17**

$$\frac{a}{3} + 1 = \frac{a}{3} + \frac{3}{3} = \frac{a+3}{3}$$

**Aufgabe 5.18**

$$7r - \frac{9}{2s} = \frac{14rs}{2s} - \frac{9}{2s} = \frac{14rs-9}{2s}$$

**Aufgabe 5.19**

$$5w - 1 + \frac{3}{w} = \frac{5w^2}{w} - \frac{w}{w} + \frac{3}{w} = \frac{5w^2 - w + 3}{w}$$

**Aufgabe 5.20**

$$8m - \frac{n}{5} = \frac{8m \cdot 5}{5} - \frac{n}{5} = \frac{40m - n}{5}$$

**Aufgabe 5.21**

$$b + \frac{1}{b} = \frac{b^2}{b} + \frac{1}{b} = \frac{b^2 + 1}{b}$$

**Aufgabe 5.22**

$$\frac{x}{4z} - 2y + 3z = \frac{x}{4z} - \frac{2y \cdot 4z}{4z} + \frac{3z \cdot 4z}{4z} = \frac{x - 8yz + 12z^2}{4z}$$

**Aufgabe 5.23**

$$\begin{aligned}3 - \frac{m}{m-n} &= \frac{3(m-n)}{m-n} - \frac{m}{m-n} = \frac{3m - 3n - m}{m-n} \\ &= \frac{2m - 3n}{m-n}\end{aligned}$$

**Aufgabe 5.24**

$$\frac{q}{q+1} - 1 = \frac{q}{q+1} - \frac{q+1}{q+1} = \frac{q - (q+1)}{q+1} = \frac{q - q - 1}{q+1} = \frac{-1}{q+1}$$

**Aufgabe 5.25**

$$\begin{aligned} e - \frac{e^2 - 2}{e - 2} &= \frac{e(e - 2)}{e - 2} - \frac{e^2 - 2}{e - 2} = \frac{e^2 - 2e - (e^2 - 2)}{e - 2} \\ &= \frac{e^2 - 2e - e^2 + 2}{e - 2} = \frac{-2e + 2}{e - 2} = \frac{2 - 2e}{e - 2} \end{aligned}$$

**Aufgabe 5.26**

$$\frac{1}{a+b} + \frac{1}{c} = \frac{c}{c(a+b)} + \frac{a+b}{c(a+b)} = \frac{a+b+c}{c(a+b)}$$

**Aufgabe 5.27**

$$\begin{aligned} \frac{8}{n+5} - \frac{n+2}{n} &= \frac{8n}{n(n+5)} - \frac{(n+2)(n+5)}{n(n+5)} \\ &= \frac{8n - (n^2 + 7n + 10)}{n(n+5)} \\ &= \frac{8n - n^2 - 7n - 10}{n(n+5)} \\ &= \frac{-n^2 + n - 10}{n(n+5)} \end{aligned}$$

**Aufgabe 5.28**

$$\begin{aligned} \frac{x+y}{x-y} - \frac{x-y}{x+y} &= \frac{(x+y)^2}{(x-y)(x+y)} - \frac{(x-y)^2}{(x+y)(x-y)} \\ &= \frac{(x+y)^2 - (x-y)^2}{(x+y)(x-y)} \\ &= \frac{(x^2 + 2xy + y^2) - (x^2 - 2xy + y^2)}{(x+y)(x-y)} \\ &= \frac{4xy}{(x+y)(x-y)} \end{aligned}$$

**Aufgabe 5.29**

$$\begin{aligned} \frac{c}{c+d} - \frac{c-d}{2(c+d)} &= \frac{2c}{2(c+d)} - \frac{c-d}{2(c+d)} \\ &= \frac{2c - (c-d)}{2(c+d)} = \frac{c+d}{2(c+d)} = \frac{1}{2} \end{aligned}$$

**Aufgabe 5.30**

$$\begin{aligned}\frac{4}{z-1} + \frac{z}{z^2-1} &= \frac{4(z+1)}{(z-1)(z+1)} + \frac{z}{(z-1)(z+1)} \\ &= \frac{4z+4+z}{(z-1)(z+1)} = \frac{5z+4}{(z-1)(z+1)} \\ &= \frac{5z+4}{z^2-1}\end{aligned}$$

**Aufgabe 5.31**

$$\begin{aligned}\frac{3u}{u^2+2uv+v^2} - \frac{1}{u+v} &= \frac{3u}{(u+v)^2} - \frac{1}{u+v} \\ &= \frac{3u}{(u+v)^2} - \frac{u+v}{(u+v)^2} \\ &= \frac{3u-(u+v)}{(u+v)^2} = \frac{2u-v}{(u+v)^2}\end{aligned}$$

**Aufgabe 5.32**

$$\begin{aligned}\frac{x-y}{15x+10y} + \frac{x+y}{3x+2y} &= \frac{x-y}{5(3x+2y)} + \frac{x+y}{3x+2y} \\ &= \frac{x-y}{5(3x+2y)} + \frac{5(x+y)}{5(3x+2y)} \\ &= \frac{x-y+5(x+y)}{5(3x+2y)} \\ &= \frac{6x+4y}{5(3x+2y)} = \frac{2(3x+2y)}{5(3x+2y)} = \frac{2}{5}\end{aligned}$$

**Aufgabe 5.33**

$$\begin{aligned}\frac{8p}{4p^2-4p+1} - \frac{3}{2p-1} &= \frac{8p}{(2p-1)^2} - \frac{3}{2p-1} \\ &= \frac{8p}{(2p-1)^2} - \frac{3(2p-1)}{(2p-1)^2} \\ &= \frac{8p-3(2p-1)}{(2p-1)^2} \\ &= \frac{8p-6p+3}{(2p-1)^2} = \frac{2p+3}{(2p-1)^2}\end{aligned}$$

**Aufgabe 5.34**

$$\begin{aligned}\frac{c}{c-d} - \frac{2cd}{c^2-d^2} - \frac{d}{c+d} \\ &= \frac{c}{c-d} - \frac{2cd}{(c-d)(c+d)} - \frac{d}{c+d} \\ &= \frac{c(c+d)}{(c-d)(c+d)} - \frac{2cd}{(c-d)(c+d)} - \frac{d(c-d)}{(c-d)(c+d)} \\ &= \frac{(c^2+cd) - 2cd - (cd-d^2)}{(c-d)(c+d)} \\ &= \frac{c^2 - 2cd + d^2}{(c-d)(c+d)} = \frac{(c-d)^2}{(c-d)(c+d)} = \frac{c-d}{c+d}\end{aligned}$$

**Aufgabe 5.35**

$$\begin{aligned}\frac{1}{a-2} + \frac{1}{a+5} - \frac{2a+3}{a^2+3a-10} \\ &= \frac{1}{a-2} + \frac{1}{a+5} - \frac{2a+3}{(a-2)(a+5)} \\ &= \frac{a+5}{(a-2)(a+5)} + \frac{a-2}{(a-2)(a+5)} - \frac{2a+3}{(a-2)(a+5)} \\ &= \frac{(a+5) + (a-2) - (2a+3)}{(a-2)(a+5)} = \frac{a+5+a-2-2a-3}{(a-2)(a+5)} \\ &= \frac{0}{(a-2)(a+5)} = 0\end{aligned}$$

**Aufgabe 5.36**

$$\begin{aligned}\frac{a-b}{4a+4b} + \frac{a+4b}{6a+6b} &= \frac{a-b}{4(a+b)} + \frac{a+4b}{6(a+b)} \\ &= \frac{3(a-b)}{12(a+b)} + \frac{2(a+4b)}{12(a+b)} \\ &= \frac{3(a-b) + 2(a+4b)}{12(a+b)} \\ &= \frac{3a - 3b + 2a + 8b}{12(a+b)} \\ &= \frac{5a + 5b}{12(a+b)} = \frac{5(a+b)}{12(a+b)} = \frac{5}{12}\end{aligned}$$

**Aufgabe 5.37**

$$\begin{aligned}\frac{t+7}{3t-6} - \frac{t+4}{t^2-2t} &= \frac{t+7}{3(t-2)} - \frac{t+4}{t(t-2)} \\ &= \frac{t(t+7)}{3t(t-2)} - \frac{3(t+4)}{3t(t-2)} \\ &= \frac{t(t+7) - 3(t+4)}{3t(t-2)} \\ &= \frac{t^2 + 7t - 3t - 12}{3t(t-2)} \\ &= \frac{t^2 + 4t - 12}{3t(t-2)} = \frac{(t-2)(t+6)}{3t(t-2)} \\ &= \frac{t+6}{3t}\end{aligned}$$

**Aufgabe 5.38**

$$\begin{aligned}\frac{u}{uv+v^2} - \frac{v}{u^2+uv} &= \frac{u}{v(u+v)} - \frac{v}{u(u+v)} \\ &= \frac{u^2}{uv(u+v)} - \frac{v^2}{uv(u+v)} \\ &= \frac{u^2 - v^2}{uv(u+v)} = \frac{(u-v)(u+v)}{uv(u+v)} \\ &= \frac{u-v}{uv}\end{aligned}$$

**Aufgabe 5.39**

$$\begin{aligned}\frac{c}{c^2-8c+16} + \frac{2}{c^2-6c+8} &= \frac{c}{(c-4)^2} + \frac{2}{(c-2)(c-4)} \\ &= \frac{c(c-2)}{(c-2)(c-4)^2} + \frac{2(c-4)}{(c-2)(c-4)^2} \\ &= \frac{c^2 - 2c + 2c - 8}{(c-2)(c-4)^2} \\ &= \frac{c^2 - 8}{(c-2)(c-4)^2}\end{aligned}$$

**Aufgabe 5.40**

$$\begin{aligned}\frac{1}{rx+ry} + \frac{1}{sx+sy} &= \frac{1}{r(x+y)} + \frac{1}{s(x+y)} \\ &= \frac{s}{rs(x+y)} + \frac{r}{rs(x+y)} \\ &= \frac{r+s}{rs(x+y)}\end{aligned}$$



**Aufgabe 5.41**

$$\begin{aligned}
\frac{a}{(a-b)(a+b)} + \frac{b}{(a-b)^2} &= \frac{a(a-b)}{(a-b)^2(a+b)} + \frac{b(a+b)}{(a-b)^2(a+b)} \\
&= \frac{a^2 - ab}{(a-b)^2(a+b)} + \frac{ab + b^2}{(a-b)^2(a+b)} \\
&= \frac{a^2 - ab + ab + b^2}{(a-b)^2(a+b)} \\
&= \frac{a^2 + b^2}{(a-b)^2(a+b)}
\end{aligned}$$

**Aufgabe 5.42**

$$\begin{aligned}
\frac{z+9}{z^2-1} - \frac{z+5}{z^2+z} &= \frac{z+9}{(z-1)(z+1)} - \frac{z+5}{z(z+1)} \\
&= \frac{z(z+9)}{z(z-1)(z+1)} - \frac{(z-1)(z+5)}{z(z-1)(z+1)} \\
&= \frac{z^2+9z}{z(z-1)(z+1)} - \frac{z^2+4z-5}{z(z-1)(z+1)} \\
&= \frac{z^2+9z - (z^2+4z-5)}{z(z-1)(z+1)} \\
&= \frac{5z+5}{z(z-1)(z+1)} \\
&= \frac{5(z+1)}{z(z-1)(z+1)} = \frac{5}{z(z-1)}
\end{aligned}$$

**Aufgabe 5.43**

$$\begin{aligned}
\frac{5}{n^2+n-6} - \frac{3}{n^2-n-2} &= \frac{5}{(n-2)(n+3)} - \frac{3}{(n-2)(n+1)} \\
&= \frac{5(n+1) - 3(n+3)}{(n-2)(n+3)(n+1)} \\
&= \frac{5n+5-3n-9}{(n-2)(n+3)(n+1)} \\
&= \frac{2n-4}{(n-2)(n+3)(n+1)} \\
&= \frac{2(n-2)}{(n-2)(n+3)(n+1)} \\
&= \frac{2}{(n+3)(n+1)}
\end{aligned}$$

**Aufgabe 5.44**

$$\frac{7}{e-1} + \frac{6}{1-e} = \frac{7}{e-1} + \frac{-6}{e-1} = \frac{7+(-6)}{e-1} = \frac{1}{e-1}$$

**Aufgabe 5.45**

$$\begin{aligned}
\frac{5}{3h-3} - \frac{4}{2-2h} &= \frac{5}{3(h-1)} - \frac{4}{-2(h-1)} \\
&= \frac{10}{6(h-1)} - \frac{-12}{6(h-1)} \\
&= \frac{10 - (-12)}{6(h-1)} = \frac{22}{6(h-1)} \\
&= \frac{11}{3(h-1)}
\end{aligned}$$

e

**Aufgabe 5.46**

$$\begin{aligned}
\frac{r-4}{5r+5} + \frac{2}{1-r^2} &= \frac{r-4}{5(r+1)} - \frac{2}{r^2-1} \\
&= \frac{r-4}{5(r+1)} - \frac{2}{(r-1)(r+1)} \\
&= \frac{(r-4)(r-1)}{5(r+1)(r-1)} - \frac{10}{5(r-1)(r+1)} \\
&= \frac{r^2-5r+4}{5(r+1)(r-1)} - \frac{10}{5(r-1)(r+1)} \\
&= \frac{r^2-5r+4-10}{5(r+1)(r-1)} = \frac{r^2-5r-6}{5(r+1)(r-1)} \\
&= \frac{(r-6)(r+1)}{5(r+1)(r-1)} = \frac{r-6}{5(r-1)}
\end{aligned}$$

**Aufgabe 5.47**

$$\begin{aligned}
\frac{2n-11}{3n-5} - \frac{4n+15}{n+7} + 1 &= \frac{(2n-11)(n+7)}{(3n-5)(n+7)} - \frac{(4n+15)(3n-5)}{(3n-5)(n+7)} + \frac{(3n-5)(n+7)}{(3n-5)(n+7)} \\
&= \frac{2n^2+3n-77}{(3n-5)(n+7)} - \frac{12n^2+25n-75}{(3n-5)(n+7)} + \frac{3n^2+16n-35}{(3n-5)(n+7)} \\
&= \frac{-7n^2-6n-37}{(3n-5)(n+7)}
\end{aligned}$$

**Aufgabe 5.48**

$$\begin{aligned}
& \frac{5}{4x-8y} - \frac{3}{10y-5x} - \frac{11}{6x-12y} \\
&= \frac{5}{4(x-2y)} - \frac{-3}{5(x-2y)} - \frac{11}{6(x-2y)} \\
&= \frac{75}{60(x-2y)} - \frac{-36}{60(x-2y)} - \frac{110}{60(x-2y)} \\
&= \frac{75 - (-36) - 110}{60(x-2y)} = \frac{1}{60(x-2y)}
\end{aligned}$$

**Aufgabe 5.49**

$$\begin{aligned}
& \frac{k+2}{6k-15} + \frac{8k+1}{8k-20} + \frac{k+11}{10-4k} \\
&= \frac{k+2}{3(2k-5)} + \frac{8k+1}{4(2k-5)} + \frac{k+11}{-2(2k-5)} \\
&= \frac{4(k+2)}{12(2k-5)} + \frac{3(8k+1)}{12(2k-5)} + \frac{-6(k+11)}{12(2k-5)} \\
&= \frac{(4k+8) + (24k+3) + (-6k-66)}{12(2k-5)} \\
&= \frac{22k-55}{12(2k-5)} = \frac{11(2k-5)}{12(2k-5)} = \frac{11}{12}
\end{aligned}$$

**Aufgabe 5.50**

$$\begin{aligned}
& \frac{1}{z^2-z} - \frac{2}{z^2} + \frac{1}{z^2+z} \\
&= \frac{1}{z(z-1)} - \frac{2}{z^2} + \frac{1}{z(z+1)} \\
&= \frac{z(z+1)}{z^2(z-1)(z+1)} - \frac{2(z-1)(z+1)}{z^2(z-1)(z+1)} + \frac{z(z-1)}{z^2(z-1)(z+1)} \\
&= \frac{z(z+1) - 2(z-1)(z+1) + z(z-1)}{z^2(z-1)(z+1)} \\
&= \frac{z^2+z-2(z^2-1)+z^2-z}{z^2(z-1)(z+1)} \\
&= \frac{2z^2-2z^2+2}{z^2(z-1)(z+1)} = \frac{2}{z^2(z^2-1)}
\end{aligned}$$

### Aufgabe 6.1

$$a \cdot \frac{b}{c} = \frac{a}{1} \cdot \frac{b}{c} = \frac{a \cdot b}{1 \cdot c} = \frac{ab}{c}$$

### Aufgabe 6.2

$$a \cdot \frac{-b}{c} = \frac{-ab}{c}$$

### Aufgabe 6.3

$$u \cdot \frac{u}{v} = \frac{u \cdot u}{v} = \frac{u^2}{v}$$

### Aufgabe 6.4

$$n \cdot \frac{m}{n^2} = \frac{n \cdot m}{n^2} = \frac{m}{n}$$

### Aufgabe 6.5

$$r^2 \cdot \frac{1}{rs} = \frac{r^2}{rs} = \frac{r}{s}$$

### Aufgabe 6.6

$$pq \cdot \frac{p}{q} = \frac{p^2 q}{q} = p^2$$

### Aufgabe 6.7

$$6ab \cdot \frac{9a}{4b} = \frac{6ab \cdot 9a}{4b} = \frac{3a \cdot 9a}{2} = \frac{27a^2}{2}$$

### Aufgabe 6.8

$$\begin{aligned} 44x^2y^2 \cdot \frac{2x^3}{11y^3} &= \frac{44x^2y^2}{1} \cdot \frac{2x^3}{11y^3} = \frac{44x^2y^2 \cdot 2x^3}{11y^3} \\ &= \frac{4x^2 \cdot 2x^3}{y} = \frac{8x^5}{y} \end{aligned}$$

### Aufgabe 6.9

$$\begin{aligned} \frac{5rs^2}{18uv^3} \cdot (-15rsuv) &= \frac{5rs^2 \cdot (-15rsuv)}{18uv^3} \\ &= \frac{-5 \cdot 15r^2s^3uv}{18uv^3} = \frac{-25r^2s^3}{6v^2} \end{aligned}$$

### Aufgabe 6.10

$$(-4pz) \left( -\frac{3q^2z}{10p^2} \right) = \frac{4pz \cdot 3q^2z}{10p^2} = \frac{2z \cdot 3q^2z}{5p} = \frac{6q^2z^2}{5p}$$

**Aufgabe 6.11**

$$(3x + 3y) \cdot \frac{9c}{x + y} = \frac{3(x + y)}{1} \cdot \frac{9c}{x + y} = \frac{3(x + y) \cdot 9c}{x + y} = 27c$$

**Aufgabe 6.12**

$$\frac{5}{q^2 - 1} \cdot (q - 1) = \frac{5(q - 1)}{(q - 1)(q + 1)} = \frac{5}{q + 1}$$

**Aufgabe 6.13**

$$\frac{d}{d^2 - 8d + 15} \cdot (d - 5) = \frac{d(d - 5)}{(d - 3)(d - 5)} = \frac{d}{d - 3}$$

**Aufgabe 6.14**

$$(2k - 7) \cdot \frac{k}{7 - 2k} = -(7 - 2k) \cdot \frac{k}{7 - 2k} = -\frac{(7 - 2k)k}{7 - 2k} = -\frac{k}{1} = -k$$

**Aufgabe 6.15**

$$\begin{aligned} (3g - 3f) \frac{4f + 4g}{5f - 5g} &= \frac{(3g - 3f)(4f + 4g)}{5f - 5g} \\ &= \frac{-3(f - g) \cdot 4(f + g)}{5(f - g)} \\ &= \frac{-12(f + g)}{5} \end{aligned}$$

**Aufgabe 6.16**

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} = \frac{ac}{bd}$$

**Aufgabe 6.17**

$$\frac{a}{b} \cdot \frac{b}{a} = \frac{ab}{ba} = 1$$

**Aufgabe 6.18**

$$\left(\frac{a}{b}\right)^3 = \frac{a}{b} \cdot \frac{a}{b} \cdot \frac{a}{b} = \frac{a^3}{b^3}$$

**Aufgabe 6.19**

$$\frac{a}{b} \cdot \frac{c}{-d} = \frac{ac}{-bd} = -\frac{ac}{bd}$$

**Aufgabe 6.20**

$$\frac{a}{-b} \cdot \frac{-c}{d} = \frac{-ac}{-bd} = \frac{ac}{bd}$$

**Aufgabe 6.21**

$$\left(-\frac{1}{n}\right)^5 = -\left(\frac{1}{n}\right)^5 = -\frac{1}{n^5}$$

**Aufgabe 6.22**

$$\frac{8a}{3b} \cdot \frac{9bc}{4a} = \frac{8a \cdot 9bc}{3b \cdot 4a} = \frac{2 \cdot 3c}{1} = 6c$$

**Aufgabe 6.23**

$$\frac{-xy^2}{35z^3} \cdot \frac{7z^2}{x^2y^2} = \frac{-xy^2 \cdot 7z^2}{35z^3 \cdot x^2y^2} = \frac{-1}{5xz}$$

**Aufgabe 6.24**

$$\frac{7m^2}{12n^3} \cdot \frac{-3n^2}{14m} = \frac{-7m^2 \cdot 3n^2}{12n^3 \cdot 14m} = \frac{-m}{8n}$$

**Aufgabe 6.25**

$$\frac{-a}{b} \cdot \frac{-b}{c} \cdot \frac{-c}{a} = \frac{(-a)(-b)(-c)}{bca} = \frac{-abc}{abc} = -1$$

**Aufgabe 6.26**

$$\left(\frac{-12}{n^3}\right)^2 = \frac{(-12)^2}{(n^3)^2} = \frac{144}{n^6}$$

**Aufgabe 6.27**

$$\left(-\frac{xyz}{cd}\right)^2 = \frac{-xyz}{cd} \cdot \frac{-xyz}{cd} = \frac{x^2y^2z^2}{c^2d^2}$$

**Aufgabe 6.28**

$$\left(\frac{-8h}{9}\right)^2 = \frac{(-8h)^2}{9^2} = \frac{64h^2}{81}$$

**Aufgabe 6.29**

$$\frac{m-n}{3m} \cdot \frac{5m}{2m-2n} = \frac{(m-n) \cdot 5m}{3m \cdot 2(m-n)} = \frac{5m}{6m} = \frac{5}{6}$$

**Aufgabe 6.30**

$$\frac{d-1}{18d} \cdot \frac{12d^2}{1-d} = \frac{(d-1) \cdot 12d^2}{18d \cdot (-1)(d-1)} = \frac{12d^2}{-18d} = \frac{-2d}{3}$$

**Aufgabe 6.31**

$$\begin{aligned} \frac{t}{4u+4v} \cdot \frac{3u^2-3v^2}{t^2+t} &= \frac{t}{4(u+v)} \cdot \frac{3(u-v)(u+v)}{t(t+1)} \\ &= \frac{1}{4} \cdot \frac{3(u-v)}{t+1} = \frac{3(u-v)}{4(t+1)} \end{aligned}$$

**Aufgabe 6.32**

$$\begin{aligned} \frac{5a^2}{5b-3} \cdot \frac{9-15b}{10ac} &= \frac{5a^2 \cdot 3(3-5b)}{(5b-3)10ac} = \frac{-15a^2(5b-3)}{10ac(5b-3)} \\ &= \frac{-15a^2}{10ac} = \frac{-3a}{2c} \end{aligned}$$

**Aufgabe 6.33**

$$\frac{p^2-q^2}{p^2+q^2} \cdot \frac{p+q}{p-q} = \frac{(p-q)(p+q) \cdot (p+q)}{(p^2+q^2) \cdot (p-q)} = \frac{(p+q)^2}{p^2+q^2}$$

*kann nicht gekürzt werden!*

**Aufgabe 6.34**

$$\begin{aligned} \frac{v^2+4v+4}{3t-3} \cdot \frac{9-9t}{v^2+5v+6} &= \frac{(v+2)^2 \cdot 9(1-t)}{-3(1-t) \cdot (v+2)(v+3)} \\ &= \frac{(v+2) \cdot 3}{(-1)(v+3)} = \frac{-3(v+2)}{v+3} \end{aligned}$$

**Aufgabe 6.35**

$$xy \left( \frac{x}{y} + \frac{y}{x} \right) = xy \cdot \frac{x}{y} + xy \cdot \frac{y}{x} = \frac{xy \cdot x}{y} + \frac{xy \cdot y}{x} = x^2 + y^2$$

**Aufgabe 6.36**

$$\begin{aligned} (n-z) \left( \frac{n}{n-z} - \frac{z}{n^2-z^2} \right) &= \frac{n-z}{1} \cdot \frac{n}{n-z} - \frac{n-z}{1} \cdot \frac{z}{(n-z)(n+z)} \\ &= n - \frac{z}{n+z} \quad \left[ = \frac{n^2+nz-z}{n+z} \right] \end{aligned}$$

**Aufgabe 6.37**

$$\left( c - \frac{d}{c} \right) \left( c + \frac{d}{c} \right) = c^2 - \left( \frac{d}{c} \right)^2 = c^2 - \frac{d^2}{c^2} \left[ = \frac{c^4 - d^2}{c^2} \right]$$

**Aufgabe 6.38**

$$\left(\frac{n}{2} - \frac{1}{n}\right)^2 = \left(\frac{n^2}{2n} - \frac{2}{2n}\right)^2 = \left(\frac{n^2 - 2}{2n}\right)^2 = \frac{(n^2 - 2)^2}{4n^2}$$

### Aufgabe 6.39

$$\begin{aligned} \left(\frac{z^2}{x-z} + z\right)^2 &= \left(\frac{z^2}{x-z} + \frac{z(x-z)}{x-z}\right)^2 \\ &= \left(\frac{z^2 + xz - z^2}{x-z}\right)^2 = \left(\frac{xz}{x-z}\right)^2 = \frac{x^2 z^2}{(x-z)^2} \end{aligned}$$

### Aufgabe 6.40

$$\begin{aligned} &\left(\frac{x}{3} - \frac{y}{2}\right)\left(\frac{x}{2} + y\right) - \left(\frac{x}{3} + y\right)\left(\frac{x}{2} - y\right) \\ &= \frac{2x - 3y}{6} \cdot \frac{x + 2y}{2} - \frac{x + 3y}{3} \cdot \frac{x - 2y}{2} \\ &= \frac{(2x - 3y)(x + 2y)}{12} - \frac{(x + 3y)(x - 2y)}{6} \\ &= \frac{2x^2 + xy - 6y^2}{12} - \frac{x^2 + xy - 6y^2}{6} \\ &= \frac{2x^2 + xy - 6y^2}{12} - \frac{2x^2 + 2xy - 12y^2}{12} = \frac{6y^2 - xy}{12} \end{aligned}$$

### Aufgabe 6.41

$$\frac{a}{b} : c = \frac{a}{b} : \frac{c}{1} = \frac{a}{b} \cdot \frac{1}{c} = \frac{a \cdot 1}{b \cdot c} = \frac{a}{bc}$$

### Aufgabe 6.42

$$\frac{-a}{b} : c = \frac{-a}{b} : \frac{c}{1} = \frac{-a \cdot 1}{b \cdot c} = \frac{-a}{bc}$$

### Aufgabe 6.43

$$\frac{mn}{d} : m = \frac{mn}{d} : \frac{m}{1} = \frac{mn}{d} \cdot \frac{1}{m} = \frac{mn}{dm} = \frac{n}{d}$$

### Aufgabe 6.44

$$\frac{u^2}{v^2} : u = \frac{u^2}{v^2} : \frac{u}{1} = \frac{u^2}{v^2 \cdot u} = \frac{u}{v^2}$$

### Aufgabe 6.45

$$\begin{aligned} \frac{7u^3}{9v^2} : 21u^2v &= \frac{7u^3}{9v^2} \cdot \frac{1}{21u^2v} = \frac{7u^3}{9v^2 \cdot 21u^2v} \\ &= \frac{u}{9v^2 \cdot 3v} = \frac{u}{27v^3} \end{aligned}$$



**Aufgabe 6.46**

$$\frac{10k - 15}{12k} : 5 = \frac{5(2k - 3)}{12k} \cdot \frac{1}{5} = \frac{5(2k - 3)}{12k \cdot 5} = \frac{2k - 3}{12k}$$

**Aufgabe 6.47**

$$\begin{aligned} \frac{3u^2v - 4uv^2}{3u + 4v} : uv &= \frac{uv(3u - 4v)}{(3u + 4v)} : \frac{uv}{1} = \frac{uv(3u - 4v)}{(3u + 4v)} \cdot \frac{1}{uv} \\ &= \frac{uv(3u - 4v)}{(3u + 4v)uv} = \frac{3u - 4v}{3u + 4v} \end{aligned}$$

**Aufgabe 6.48**

$$\frac{q^3 + q^2}{4} : q^3 = \frac{q^2(q + 1)}{4q^3} = \frac{q + 1}{4q}$$

**Aufgabe 6.49**

$$\begin{aligned} \frac{2p - 8}{15} : (4 - p) &= \frac{-2(4 - p)}{15} : \frac{4 - p}{1} \\ &= \frac{-2(4 - p)}{15} \cdot \frac{1}{4 - p} \\ &= \frac{-2(4 - p)}{15(4 - p)} = \frac{-2}{15} \end{aligned}$$

**Aufgabe 6.50**

$$\begin{aligned} \frac{x + y}{x - y} : (x^2 - y^2) &= \frac{x + y}{x - y} : \frac{x^2 - y^2}{1} \\ &= \frac{x + y}{x - y} \cdot \frac{1}{(x - y)(x + y)} \\ &= \frac{x + y}{(x - y)(x - y)(x + y)} = \frac{1}{(x - y)^2} \end{aligned}$$

**Aufgabe 6.51**

$$\frac{a}{b} : \left(-\frac{c}{d}\right) = \frac{a}{b} : \frac{-c}{d} = \frac{a}{b} \cdot \frac{d}{-c} = \frac{ad}{-bc} = -\frac{ad}{bc}$$

**Aufgabe 6.52**

$$\frac{a}{b} : \frac{a}{b} = \frac{a}{b} \cdot \frac{b}{a} = \frac{ab}{ba} = 1$$

**Aufgabe 6.53**

$$\frac{5km}{6} : \frac{3k}{2m} = \frac{5km}{6} \cdot \frac{2m}{3k} = \frac{5km \cdot 2m}{6 \cdot 3k} = \frac{5m \cdot m}{3 \cdot 3} = \frac{5m^2}{9}$$

**Aufgabe 6.54**

$$\frac{112n^2}{19xyz} : \frac{-7n}{19xyz} = \frac{112n^2}{19xyz} \cdot \frac{19xyz}{-7n} = \frac{112n^2 \cdot 19xyz}{19xyz \cdot (-7n)} = -16n$$

**Aufgabe 6.55**

$$\frac{1}{24rs^3} : \frac{1}{16r^2s} = \frac{1}{24rs^3} \cdot \frac{16r^2s}{1} = \frac{16r^2s}{24rs^3} = \frac{2r}{3s^2}$$

**Aufgabe 6.56**

$$\left(-\frac{78f}{85h^3}\right) : \left(-\frac{48f^2}{85h^3}\right) = \frac{-78f}{85h^3} \cdot \frac{85h^3}{-48f^2} = \frac{-78f}{-48f^2} = \frac{13}{8f}$$

**Aufgabe 6.57**

$$\begin{aligned} \frac{uv}{u+v} : \frac{5v}{u^2+uv} &= \frac{uv}{u+v} \cdot \frac{u(u+v)}{5v} \\ &= \frac{uv \cdot u(u+v)}{(u+v) \cdot 5v} = \frac{u^2}{5} \end{aligned}$$

**Aufgabe 6.58**

$$\begin{aligned} \frac{z}{3z-3} : \frac{z}{2-2z} &= \frac{z}{3(z-1)} \cdot \frac{2(1-z)}{z} \\ &= \frac{-2z(z-1)}{3z(z-1)} = -\frac{2}{3} \end{aligned}$$

**Aufgabe 6.59**

$$\frac{n^2 - 19n + 90}{n + 9} : \frac{n - 9}{n + 9} = \frac{(n - 9)(n - 10)}{n + 9} \cdot \frac{n + 9}{n - 9} = n - 10$$

**Aufgabe 6.60**

$$\begin{aligned} \frac{c^2 - d^2}{c - 1} : \frac{c + d}{1 - c} &= \frac{(c - d)(c + d)}{c - 1} \cdot \frac{1 - c}{c + d} \\ &= \frac{(c - d)(c + d)(1 - c)}{(c - 1)(c + d)} \\ &= \frac{(c - d)(1 - c)}{c - 1} \\ &= \frac{-(c - d)(c - 1)}{c - 1} = d - c \end{aligned}$$

**Aufgabe 6.61**

$$\begin{aligned} \frac{w^2 - w - 12}{t^2} : \frac{w - 4}{t^2 - t} &= \frac{(w + 3)(w - 4)}{t^2} \cdot \frac{t(t - 1)}{w - 4} \\ &= \frac{(w + 3)(w - 4) \cdot t(t - 1)}{t^2 \cdot (w - 4)} \\ &= \frac{(w + 3)(t - 1)}{t} \end{aligned}$$

**Aufgabe 6.62**

$$\begin{aligned} \frac{196a^2 - 25}{4b^2 + 20b + 25} : \frac{70a + 25}{2b + 5} &= \frac{(14a - 5)(14a + 5)}{(2b + 5)^2} \cdot \frac{2b + 5}{5(14a + 5)} \\ &= \frac{14a - 5}{5(2b + 5)} \end{aligned}$$

**Aufgabe 6.62**

$$\begin{aligned} \frac{a^3 + a^2b}{c^2 + 1} : \frac{a^3 - ab^2}{c^2 - c} &= \frac{a^2(a + b)}{c^2 + 1} \cdot \frac{c(c - 1)}{a(a^2 - b^2)} \\ &= \frac{a^2(a + b) \cdot c(c - 1)}{(c^2 + 1) \cdot a(a - b)(a + b)} \\ &= \frac{ac(c - 1)}{(a - b)(c^2 + 1)} \end{aligned}$$

**Aufgabe 6.63**

$$\begin{aligned} \frac{10x^2 - 20x + 10}{9x^2 + 18x + 9} : \frac{15x^2 + 15x - 30}{2x^2 - 2x - 4} &= \frac{10(x^2 - 2x + 1)}{9(x^2 + 2x + 1)} : \frac{15(x^2 + x - 2)}{2(x^2 - x - 2)} \\ &= \frac{10(x - 1)^2}{9(x + 1)^2} : \frac{15(x - 1)(x + 2)}{2(x + 1)(x - 2)} \\ &= \frac{10(x - 1)^2}{9(x + 1)^2} \cdot \frac{2(x + 1)(x - 2)}{15(x - 1)(x + 2)} \\ &= \frac{10(x - 1)^2 \cdot 2(x + 1)(x - 2)}{9(x + 1)^2 \cdot 15(x - 1)(x + 2)} \\ &= \frac{4(x - 1)(x - 2)}{27(x + 1)(x + 2)} \end{aligned}$$

**Aufgabe 6.64**

$$(u + v) : \frac{u + v}{w} = \frac{u + v}{1} \cdot \frac{w}{u + v} = \frac{(u + v)w}{u + v} = w$$

**Aufgabe 6.65**

$$(p+q) : \frac{p^2 - q^2}{pq} = (p+q) \cdot \frac{pq}{(p-q)(p+q)} = \frac{pq}{p-q}$$

**Aufgabe 6.66**

$$\begin{aligned}(6d^2 - 9d) : \frac{4d - 6}{2d + 3} &= \frac{3d(2d - 3)}{1} \cdot \frac{2d + 3}{2(2d - 3)} \\ &= \frac{3d(2d - 3) \cdot (2d + 3)}{2(2d - 3)} = \frac{3d(2d + 3)}{2}\end{aligned}$$

**Aufgabe 6.67**

$$\begin{aligned}\left(4ef - \frac{2e}{f}\right) : \frac{2e}{f} &= \left(\frac{4ef^2}{f} - \frac{2e}{f}\right) \cdot \frac{f}{2e} \\ &= \frac{4ef^2 - 2e}{f} \cdot \frac{f}{2e} \\ &= \frac{2e \cdot (2f^2 - 1) \cdot f}{f \cdot 2e} = 2f^2 - 1\end{aligned}$$

**Aufgabe 6.68**

$$\begin{aligned}\left(\frac{a}{b} - \frac{c}{d}\right) : \left(\frac{a}{b} + \frac{c}{d}\right) &= \left(\frac{ad}{bd} - \frac{bc}{bd}\right) : \left(\frac{ad}{bd} + \frac{bc}{bd}\right) \\ &= \frac{ad - bc}{bd} : \frac{ad + bc}{bd} \\ &= \frac{ad - bc}{bd} \cdot \frac{bd}{ad + bc} = \frac{ad - bc}{ad + bc}\end{aligned}$$

**Aufgabe 6.69**

$$\begin{aligned}\left(x - \frac{1}{x}\right) : \left(x + \frac{1}{x}\right) &= \left(\frac{x^2}{x} - \frac{1}{x}\right) : \left(\frac{x^2}{x} + \frac{1}{x}\right) \\ &= \frac{x^2 - 1}{x} : \frac{x^2 + 1}{x} \\ &= \frac{x^2 - 1}{x} \cdot \frac{x}{x^2 + 1} \\ &= \frac{(x^2 - 1)x}{x(x^2 + 1)} = \frac{x^2 - 1}{x^2 + 1}\end{aligned}$$

### Aufgabe 7.1

$$\left(\frac{2a+1}{a} - 1\right)^2 = \left(\frac{2a+1}{a} - \frac{a}{a}\right)^2 = \left(\frac{a+1}{a}\right)^2 = \frac{(a+1)^2}{a^2}$$

### Aufgabe 7.2

$$\begin{aligned} 4y^2z^3 \left(\frac{2x}{yz^2} - \frac{3x}{y^2z}\right) &: (3z - 2y) \\ &= 4y^2z^3 \left(\frac{2xy}{y^2z^2} - \frac{3xz}{y^2z^2}\right) : (3z - 2y) \\ &= 4y^2z^3 \cdot \frac{(2xy - 3xz)}{y^2z^2} \cdot \frac{1}{3z - 2y} \\ &= \frac{4y^2z^3 \cdot (-x) \cdot (3z - 2y)}{y^2z^2 \cdot (3z - 2y)} = -4xz \end{aligned}$$

### Aufgabe 7.3

$$\begin{aligned} u^2v^2 \left(\frac{u}{v} - \frac{v}{u}\right)^2 &: (u-v)(u+v)^2 \\ &= u^2v^2 \left(\frac{u^2 - v^2}{uv}\right)^2 : (u-v)(u+v)^2 \\ &= u^2v^2 \left(\frac{u^2 - v^2}{uv}\right)^2 \cdot \frac{1}{(u-v)(u+v)^2} \\ &= u^2v^2 \left[\frac{(u-v)(u+v)}{uv}\right]^2 \cdot \frac{1}{(u-v)(u+v)^2} \\ &= u^2v^2 \frac{(u-v)^2(u+v)^2}{u^2v^2} \cdot \frac{1}{(u-v)(u+v)^2} = u - v \end{aligned}$$

### Aufgabe 7.4

$$\begin{aligned} \left(\frac{7ab}{5c-5d} \cdot \frac{4e^3}{9f^3}\right) &: \frac{14b}{3c-3d} \cdot \left(\frac{2e}{3f}\right)^2 \\ &= \frac{7ab}{5(c-d)} \cdot \frac{4e^3}{9f^3} \cdot \frac{3(c-d)}{14b} \cdot \frac{4e^2}{9f^2} \\ &= \frac{8ae^5}{135f^5} \end{aligned}$$

**Aufgabe 7.5**

$$\begin{aligned} & \frac{25x^2 - 9}{(x+2)^2} \cdot \frac{x^2 + 5x + 6}{y^3} : \frac{5x-3}{xy^3 + 2y^3} \\ &= \frac{(5x+3)(5x-3)}{(x+2)^2} \cdot \frac{(x+2)(x+3)}{y^3} \cdot \frac{y^3(x+2)}{5x-3} \\ &= \frac{(5x+3)(5x-3)(x+2)(x+3)y^3(x+2)}{(x+2)^2y^3(5x-3)} \\ &= (5x+3)(x+3) \end{aligned}$$

**Aufgabe 7.6**

$$(c-5) : \frac{c^2 - 3c - 10}{c^2 - 4} = (c-5) \cdot \frac{(c-2)(c+2)}{(c-5)(c+2)} = c-2$$

**Aufgabe 7.7**

$$\begin{aligned} & \left( \frac{2}{m-1} + m + 1 \right) \cdot \left( \frac{1}{m^2-1} - \frac{2m}{m^4-1} \right) \\ &= \left( \frac{2}{m-1} + \frac{(m+1)(m-1)}{m-1} \right) \cdot \left( \frac{1}{m^2-1} - \frac{2m}{(m^2-1)(m^2+1)} \right) \\ &= \frac{2+m^2-1}{m-1} \cdot \left( \frac{m^2+1}{(m^2-1)(m^2+1)} - \frac{2m}{(m^2-1)(m^2+1)} \right) \\ &= \frac{m^2+1}{m-1} \cdot \frac{m^2-2m+1}{(m^2-1)(m^2+1)} = \frac{m^2+1}{m-1} \cdot \frac{(m-1)^2}{(m^2-1)(m^2+1)} \\ &= \frac{m-1}{m^2-1} = \frac{m-1}{(m-1)(m+1)} = \frac{1}{m+1} \end{aligned}$$

### Aufgabe 8.1

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} : \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

### Aufgabe 8.2

$$\frac{\frac{u}{v}}{x} = \frac{u}{v} : x = \frac{u}{vx}$$

### Aufgabe 8.3

$$\frac{\frac{u}{u}}{\frac{v}{v}} = \frac{u}{1} : \frac{u}{v} = \frac{u}{1} \cdot \frac{v}{u} = v$$

### Aufgabe 8.4

$$\frac{\frac{14u^3v}{57xyz}}{\frac{35uv^2}{76y^2z}} = \frac{14u^3v}{57xyz} \cdot \frac{76y^2z}{35uv^2} = \frac{2 \cdot 7 \cdot u^2 \cdot 4 \cdot 19 \cdot y}{3 \cdot 19 \cdot x \cdot 5 \cdot 7 \cdot v} = \frac{8u^2y}{15xv}$$

### Aufgabe 8.5

$$\begin{aligned} \frac{\frac{1}{x} + \frac{1}{y}}{\frac{x}{y} - \frac{y}{x}} &= \left( \frac{1}{x} + \frac{1}{y} \right) : \left( \frac{x}{y} - \frac{y}{x} \right) = \frac{y+x}{xy} : \frac{x^2-y^2}{xy} \\ &= \frac{x+y}{xy} \cdot \frac{xy}{(x-y)(x+y)} = \frac{1}{x-y} \end{aligned}$$

### Aufgabe 8.6

$$\begin{aligned} & \frac{\frac{x-y}{x+y} - \frac{x}{x-y}}{\frac{x+y}{x-y} - \frac{y}{x+y}} \\ &= \left( \frac{x-y}{x+y} - \frac{x}{x-y} \right) : \left( \frac{x+y}{x-y} - \frac{y}{x+y} \right) \\ &= \left[ \frac{(x-y)(x-y)}{(x+y)(x-y)} - \frac{x(x+y)}{(x+y)(x-y)} \right] : \left[ \frac{(x+y)(x+y)}{(x+y)(x-y)} - \frac{y(x-y)}{(x+y)(x-y)} \right] \\ &= \frac{x^2 - 2xy + y^2 - x^2 - xy}{(x+y)(x-y)} : \frac{x^2 + 2xy + y^2 - xy + y^2}{(x+y)(x-y)} \\ &= \frac{y^2 - 3xy}{(x+y)(x-y)} : \frac{x^2 + xy + 2y^2}{(x+y)(x-y)} \\ &= \frac{y^2 - 3xy}{x^2 + xy + 2y^2} \end{aligned}$$

### Aufgabe 8.7

$$\begin{aligned} & \frac{\frac{n}{n^2-1}}{\frac{1}{n+1} - \frac{1}{n-1}} = \frac{n}{(n-1)(n+1)} : \left( \frac{1}{n+1} - \frac{1}{n-1} \right) \\ &= \frac{n}{(n-1)(n+1)} : \frac{(n-1) - (n+1)}{(n-1)(n+1)} \\ &= \frac{n}{(n-1)(n+1)} : \frac{-2}{(n-1)(n+1)} \\ &= \frac{n}{(n-1)(n+1)} \cdot \frac{(n-1)(n+1)}{-2} \\ &= \frac{n}{-2} = \frac{-n}{2} \end{aligned}$$



**Aufgabe 9.1**

$$\frac{1}{x} + 2 = \frac{9}{x} \quad D = \mathbb{R} \setminus \{0\} \quad || \cdot x$$

$$\frac{1}{x} \cdot x + 2 \cdot x = \frac{9}{x} \cdot x$$

$$1 + 2x = 9 \quad || - 1$$

$$2x = 8 \quad || : 2$$

$$x = 4$$

$$L = \{4\}$$

**Aufgabe 9.2**

$$\frac{5}{6x} + \frac{13}{4} = \frac{5}{3} - \frac{2}{9x} \quad D = \mathbb{R} \setminus \{0\} \quad || \cdot 36x$$

$$30 + 117x = 60x - 8 \quad || - 60x - 30$$

$$57x = -38 \quad || : 19$$

$$x = -\frac{2}{3}$$

$$L = \left\{-\frac{2}{3}\right\}$$

**Aufgabe 9.3**

$$\frac{2x-4}{x} = \frac{8x-7}{x} \quad D = \mathbb{R} \setminus \{0\} \quad || \cdot x$$

$$2x - 4 = 8x - 7$$

$$3 = 6x$$

$$x = \frac{1}{2}$$

$$L = \left\{\frac{1}{2}\right\}$$

**Aufgabe 9.4**

$$\frac{1}{x} = \frac{9}{x-5} \quad D = \mathbb{R} \setminus \{0, 5\} \quad || \cdot x(x-5)$$

$$x - 5 = 9x$$

$$-5 = 8x$$

$$x = -\frac{5}{8}$$

$$L = \left\{-\frac{5}{8}\right\}$$

### Aufgabe 9.5

$$\frac{2}{x-3} = \frac{3}{x+5} \quad D = \mathbb{R} \setminus \{3, -5\} \quad || \cdot (x-3)(x+5)$$

$$2(x+5) = 3(x-3)$$

$$2x + 10 = 3x - 9$$

$$19 = x$$

$$L = \{19\}$$

### Aufgabe 9.6

$$\frac{2x+19}{x+2} = \frac{47}{3x+6} \quad D = \mathbb{R} \setminus \{-2\} \quad || \cdot 3(x+2),$$

$$\frac{2x+19}{x+2} = \frac{47}{3(x+2)}$$

$$3(2x+19) = 47$$

$$6x + 57 = 47$$

$$6x = -10$$

$$3x = -5$$

$$x = -\frac{5}{3}$$

$$L = \left\{-\frac{5}{3}\right\}$$

### Aufgabe 9.7

$$\frac{2x}{x-5} = \frac{x-24}{5-x} \quad D = \mathbb{R} \setminus \{5\}$$

$$\frac{2x}{x-5} = \frac{24-x}{x-5} \quad || \cdot (x-5)$$

$$2x = 24 - x$$

$$3x = 24$$

$$x = 8$$

$$L = \{8\}$$

### Aufgabe 9.8

$$\frac{x+1}{2(x-1)} - \frac{5x-8}{2(x-1)} = \frac{3(x-4)}{2(x-1)} \quad D = \mathbb{R} \setminus \{1\} \quad || \cdot 2(x-1)$$

$$(x+1) - (5x-8) = 3(x-4) \quad \text{Vorsicht!}$$

$$x+1-5x+8 = 3x-12$$

$$9-4x = 3x-12$$

$$21 = 7x$$

$$x = 3$$

$$L = \{3\}$$

### Aufgabe 9.9

$$\frac{x-1}{x+1} - \frac{2x-1}{2x+2} = \frac{4x-1}{4x+4} \quad D = \mathbb{R} \setminus \{-1\}$$

$$\frac{x-1}{x+1} - \frac{2x-1}{2(x+1)} = \frac{4x-1}{4(x+1)} \quad || \cdot 4(x+1)$$

$$4(x-1) - 2(2x-1) = 4x-1 \quad \text{Vorsicht!}$$

$$4x-4-4x+2 = 4x-1$$

$$-2 = 4x-1$$

$$-1 = 4x$$

$$x = -\frac{1}{4}$$

$$L = \left\{-\frac{1}{4}\right\}$$

### Aufgabe 9.10

$$\frac{z-3}{z-2} + \frac{z}{5z-10} = \frac{4}{5} \quad D = \mathbb{R} \setminus \{2\}$$

$$\frac{z-3}{z-2} + \frac{z}{5(z-2)} = \frac{4}{5} \quad || \cdot 5(z-2)$$

$$5(z-3) + z = 4(z-2)$$

$$5z-15+z = 4z-8$$

$$2z = 7$$

$$z = \frac{7}{2}$$

$$L = \left\{\frac{7}{2}\right\}$$

**Aufgabe 9.11**

$$\frac{3}{x-2} - \frac{1}{x+2} = \frac{2x+8}{x^2-4} \quad D = \mathbb{R} \setminus \{-2, 2\}$$

$$\frac{3}{x-2} - \frac{1}{x+2} = \frac{2(x+4)}{(x-2)(x+2)} \quad || \cdot (x-2)(x+2)$$

$$3(x+2) - (x-2) = 2(x+4) \quad \text{Vorsicht!}$$

$$3x+6 - x+2 = 2x+8$$

$$2x+8 = 2x+8$$

$$0 = 0$$

$$L = \mathbb{R} \setminus \{-2, 2\} = D$$

**Aufgabe 9.12**

$$\frac{5}{n-4} - \frac{1}{n-5} = \frac{9n-1}{n^2-9n+20} \quad D = \mathbb{R} \setminus \{4, 5\}$$

$$\frac{5}{n-4} - \frac{1}{n-5} = \frac{9n-1}{(n-4)(n-5)} \quad || \cdot (n-4)(n-5)$$

$$5(n-5) - (n-4) = 9n-1$$

$$5n-25 - n+4 = 9n-1$$

$$4n-21 = 9n-1 \quad || -4n+1$$

$$-20 = 5n \quad || :5$$

$$n = -4$$

$$L = \{-4\}$$

**Aufgabe 9.13**

$$\frac{2}{x+2} - \frac{2}{x-2} = \frac{x+3}{4-x^2} \quad D = \mathbb{R} \setminus \{-2, 2\}$$

$$\frac{2}{x+2} - \frac{2}{x-2} = \frac{x+3}{(2-x)(2+x)} \quad || \cdot (x-2)(x+2)$$

$$2(x-2) - 2(x+2) = -(x+3)$$

$$2x-4 - 2x-4 = -x-3$$

$$-8 = -x-3$$

$$x = 5$$

$$L = \{5\}$$

**Aufgabe 9.14**

$$\frac{x}{3x-4} + \frac{1}{8-6x} - 2 = 0 \quad D = \mathbb{R} \setminus \left\{ \frac{4}{3} \right\}$$

$$\frac{x}{3x-4} + \frac{1}{2(4-3x)} - 2 = 0$$

$$\frac{x}{3x-4} - \frac{1}{2(3x-4)} - 2 = 0 \quad || \cdot 2(3x-4)$$

$$2x - 1 - 4(3x - 4) = 0$$

$$2x - 1 - 12x + 16 = 0$$

$$-10x + 15 = 0 \quad || + 10x$$

$$15 = 10x \quad || : 5$$

$$x = \frac{3}{2}$$

$$L = \left\{ \frac{3}{2} \right\}$$

### Aufgabe 9.15

$$\frac{x-4}{x-2} + \frac{x-8}{2x-4} + \frac{x-9}{3x-6} = 0 \quad D = \mathbb{R} \setminus \{2\}$$

$$\frac{x-4}{x-2} + \frac{x-8}{2(x-2)} + \frac{x-9}{3(x-2)} = 0 \quad || \cdot 6(x-2)$$

$$6(x-4) + 3(x-8) + 2(x-9) = 0$$

$$6x - 24 + 3x - 24 + 2x - 18 = 0$$

$$11x - 66 = 0$$

$$11x = 66$$

$$x = 6$$

$$L = \{6\}$$

**Aufgabe 9.16\***

$$\frac{x-1}{x-2} - \frac{x-5}{x-6} = \frac{x-3}{x-4} - \frac{x-7}{x-8} \quad D = \mathbb{R} \setminus \{2, 6, 3, 4\}$$

$$\frac{(x-1)(x-6) - (x-5)(x-2)}{(x-6)(x-2)} = \frac{(x-3)(x-8) - (x-7)(x-4)}{(x-8)(x-4)}$$

$$\frac{(x^2 - 7x + 6) - (x^2 - 7x + 10)}{(x-6)(x-2)} = \frac{(x^2 - 11x + 24) - (x^2 - 11x + 28)}{(x-8)(x-4)}$$

$$\frac{x^2 - 7x + 6 - x^2 + 7x - 10}{(x-6)(x-2)} = \frac{x^2 - 11x + 24 - x^2 + 11x - 28}{(x-8)(x-4)}$$

$$\frac{-4}{(x-6)(x-2)} = \frac{-4}{(x-8)(x-4)} \quad \left[ \frac{a}{b} = \frac{a}{c} \Rightarrow b = c \right]$$

$$(x-6)(x-2) = (x-8)(x-4)$$

$$x^2 - 8x + 12 = x^2 - 12x + 32$$

$$-8x + 12 = -12x + 32$$

$$4x = 20$$

$$x = 5 \quad L = \{5\}$$

**Aufgabe 9.17**

$$\frac{1}{x} = \frac{4}{9} \quad D = \mathbb{R} \setminus \{0\} \quad \left[ \frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c} \right]$$

$$\frac{x}{1} = \frac{9}{4}$$

$$L = \left\{ \frac{9}{4} \right\}$$

**Aufgabe 9.18**

$$\left( \frac{1}{x} - \frac{2}{3} \right) \cdot \left( \frac{1}{x} + 7 \right) = 0 \quad D = \mathbb{R} \setminus \{0\}$$

$$\left( \frac{2}{2x} - \frac{2}{3} \right) \cdot \left( \frac{7}{7x} + \frac{7}{1} \right) = 0$$

$$L = \left\{ \frac{3}{2}, -\frac{1}{7} \right\}$$

*Merke:* Ein Produkt ist genau dann null, wenn mindestens ein Faktor null ist.

**Aufgabe 9.19**

$$x + 1 = \frac{a}{a + b}$$

$$x = \frac{a}{a + b} - 1$$

$$x = \frac{a}{a + b} - \frac{a + b}{a + b}$$

$$x = \frac{a - (a + b)}{a + b}$$

$$x = \frac{-b}{a + b}$$

**Aufgabe 9.20**

$$\frac{r}{r - 1} = \frac{r}{r + 1} + x$$

$$\frac{r}{r - 1} - \frac{r}{r + 1} = x$$

$$x = \frac{r}{r - 1} - \frac{r}{r + 1}$$

$$x = \frac{r(r + 1)}{(r - 1)(r + 1)} - \frac{r(r - 1)}{(r - 1)(r + 1)}$$

$$x = \frac{r^2 + r - (r^2 - r)}{(r - 1)(r + 1)}$$

$$x = \frac{2r}{r^2 - 1} = \frac{2r}{r^2 - 1}$$

**Aufgabe 9.21**

$$x + \frac{x}{p} = 1 \quad || \cdot p$$

$$px + x = p$$

$$x(p + 1) = p$$

$$x = \frac{p}{p + 1}$$

**Aufgabe 9.22**

$$\frac{x}{e} + \frac{x}{f} = \frac{1}{f} \quad || \cdot ef$$

$$xf + xe = e$$

$$x(f + e) = e$$

$$x = \frac{e}{e + f}$$

### Aufgabe 9.23

$$A = \frac{abc}{4r}$$

$$\bullet a = \frac{4Ar}{bc}$$

$$\bullet b = \frac{4Ar}{ac}$$

$$\bullet c = \frac{4Ar}{ab}$$

$$\bullet r = \frac{abc}{4A}$$

### Aufgabe 9.24

$$s = \frac{1}{1-q}$$

$$s(1-q) = 1$$

$$s - sq = 1$$

$$s - 1 = sq$$

$$\frac{s-1}{s} = q$$

$$q = \frac{s-1}{s}$$

### Aufgabe 9.25

$$\frac{1}{x+y} = \frac{1}{x \cdot y}$$

$$\bullet \frac{1}{x+y} = \frac{1}{x \cdot y}$$

$$x+y = x \cdot y$$

$$x+y-xy = 0$$

$$x(1-y) = -y$$

$$x = \frac{-y}{1-y}$$

$$x = \frac{y}{y-1}$$

$$\bullet x+y-xy = 0$$

$$y(1-x) = -x$$

$$y = \frac{-x}{1-x}$$

$$y = \frac{x}{x-1}$$



### Aufgabe 9.26

$$\frac{1}{b} + \frac{1}{g} = \frac{1}{f}$$

- $\frac{1}{b} + \frac{1}{g} = \frac{1}{f} \quad || \cdot bfg$

$$gf + bf = bg$$

$$bf - bg = -gf$$

$$b(f - g) = -gf$$

$$b = \frac{-gf}{f - g}$$

$$b = \frac{gf}{g - f}$$

- $gf + bf = bg$

$$gf + bf = bg$$

$$f(g + b) = bg$$

$$f = \frac{bg}{b + g}$$

- $gf + bf = bg$

$$gf - bg = -bf$$

$$g(f - b) = -bf$$

$$g = \frac{-bf}{f - b}$$

$$g = \frac{bf}{b - f}$$

### Aufgabe 9.27

Zähler des ungekürzten Bruches:  $x$

Nenner des ungekürzten Bruches:  $x - 3$

$$\frac{x}{x - 3} = \frac{4}{5}$$

$$5x = 4(x - 3)$$

$$5x = 4x - 12$$

$$x = -12$$

Der ungekürzte Bruch lautet  $\frac{-12}{-15}$ .

### Aufgabe 9.28

Anzahl Stunden, die der kleine Bagger alleine braucht:  $x$   
Anzahl Stunden, die der grosse Bagger alleine braucht: 12

$$\text{Leistung} = \frac{\text{Arbeit}}{\text{Zeit}}$$

$$\frac{1 \text{ Aushub}}{12} + \frac{1 \text{ Aushub}}{x} = \frac{1 \text{ Aushub}}{9}$$

$$\frac{1}{12} + \frac{1}{x} = \frac{1}{9}$$

$$3x + 36 = 4x$$

$$36 = x$$

Der kleine Bagger benötigt alleine 36 Stunden für den Aushub.

### Aufgabe 9.29

Zeit für 120 km mit kleiner Geschwindigkeit:  $t$   
Zeit für 120 km mit grosser Geschwindigkeit:  $0.96 \cdot t$

$$\text{Geschwindigkeit} = \frac{\text{Weg}}{\text{Zeit}} = \frac{s}{t}$$

$$v_{\text{gross}} = v_{\text{klein}} + 4$$

$$\frac{120}{0.96 \cdot t} = \frac{120}{t} + 4 \quad || \cdot 0.96t$$

$$120 = 115.2 + 3.84t$$

$$4.8 = 3.84t$$

$$t = 1.25$$

Der Autofahrer ist 1.25 Stunden früher (um 13.45 Uhr) abgefahren.

### Aufgabe 9.30

Anzahl Fahrten des grossen Lastwagens (alleine):  $x$

Anzahl Fahrten des kleinen Lastwagens (alleine):  $x + 9$

Leistung<sub>k</sub> + Leistung<sub>g</sub> = Leistung<sub>k+g</sub>

$$\frac{1 \text{ Schutt}}{x + 9} + \frac{1 \text{ Schutt}}{x} = \frac{1 \text{ Schutt}}{20}$$

$$\frac{1}{x + 9} + \frac{1}{x} = \frac{1}{20} \quad || \cdot 20x(x + 9)$$

$$20x + 20(x + 9) = x(x + 9)$$

$$40x + 180 = x^2 + 9x$$

$$40x + 180 = x^2 + 9x$$

$$0 = x^2 - 31x - 180$$

$$0 = (x - 36)(x + 5)$$

$$x = 36 \quad \vee \quad x = -5 \text{ (sinnlos)}$$

Der grosse Lastwagen muss 36 Mal, der kleine 45 Mal fahren.