3. (a) $\sqrt{6^2+8^2} = \sqrt{36+64} = \sqrt{100} = \mathbf{10\,m}$
   (b) $\sqrt{17^2-8^2} = \sqrt{289-64} = \sqrt{225} = \mathbf{15\,dm}$
   (c) $\sqrt{5^2-3^2} = \sqrt{25-9} = \sqrt{16} = \mathbf{4\,mm}$
   (d) $\sqrt{5^2+12^2} = \sqrt{25-144} = \sqrt{169} = \mathbf{13\,cm}$

4. (a) $\sqrt{5^2+2^2} = \sqrt{25+4} = \sqrt{29} \approx \mathbf{5.39\,cm}$
   (b) $\sqrt{7^2-6^2} = \sqrt{49+36} = \sqrt{13} \approx \mathbf{3.61\,dm}$
   (c) $\sqrt{78^2+54^2} = \sqrt{6084+2916} = \sqrt{9000} \approx \mathbf{94.87\,mm}$

5. (a)
   - $x = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = \mathbf{5\,cm}$
   - $y = \sqrt{6^2-5^2} = \sqrt{36+25} = \sqrt{11} \approx \mathbf{3.32\,cm}$
   - $A = y^2 = \sqrt{11}^2 = \mathbf{11\,cm^2}$

   (b)
   - $x = \sqrt{10^2-9^2} = \sqrt{100+81} = \sqrt{19} \approx \mathbf{4.36\,dm}$
   - $A_x = x^2 = 19\,\mathrm{dm}^2$
   - $y = \sqrt{10^2-7^2} = \sqrt{100+49} = \sqrt{51} \approx \mathbf{7.14\,dm}$
   - $A_y = y^2 = \mathbf{51\,dm^2}$

7. (a) $x = \sqrt{\sqrt{6}^2+\sqrt{6}^2} = \sqrt{6+6} = \sqrt{12} \approx \mathbf{3.46\,cm}$
   (b) $y = \sqrt{\sqrt{7}^2+\sqrt{3}^2} = \sqrt{7+3} = \sqrt{10} \approx \mathbf{3.16\,cm}$
   (c) $z = \sqrt{\sqrt{7}^2-\sqrt{3}^2} = \sqrt{7-3} = \sqrt{4} = \mathbf{2\,cm}$

8. (b)
   - $x_1 = \sqrt{1^2+1^2} = \sqrt{2}$
   - $x_2 = \sqrt{\sqrt{2}^2+1^2} = \sqrt{2+1} = \sqrt{3}$
   - $x_3 = \sqrt{\sqrt{3}^2+1^2} = \sqrt{3+1} = \sqrt{4}$
   - $x_4 = \sqrt{\sqrt{4}^2+1^2} = \sqrt{4+1} = \sqrt{5}$
   - $\ldots$
   - $x_{15} = \sqrt{\sqrt{14}^2+1^2} = \sqrt{15+1} = \sqrt{16} = \mathbf{4\,cm}$

10. (a) $\sqrt{80^2+50^2} = \sqrt{6400+2500} = \sqrt{8900} \approx \mathbf{94.34\,mm}$
    (b) $\sqrt{15^2-9^2} = \sqrt{225+81} = \sqrt{144} = \mathbf{12\,dm}$
    (c) Diagonale im Quadrat: $d = \sqrt{2}\cdot a = \sqrt{2}\cdot 1 \approx \mathbf{1.41\,dm}$
    (d) Diagonale im Quadrat: $d = \sqrt{2}\cdot a \quad \Leftrightarrow \quad a = d/\sqrt{2} = 4/\sqrt{2} \approx \mathbf{2.83\,cm}$

11. (a)
    - $d = \sqrt{a^2+b^2} = \sqrt{15.2^2+8.4^2} = \sqrt{301.6} \approx \mathbf{17.37\,dm}$
    - $u = 2(a+b) = 2(15.2+8.4) = \mathbf{47.2\,dm}$
    - $A = a\cdot b = 15.2\cdot 8.4 = \mathbf{127.68\,dm^2}$

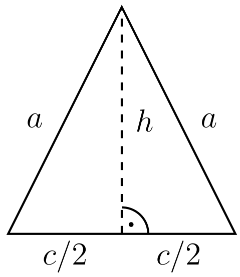
    (b)
    - $u = 2(a+b) \Rightarrow u/2 = a+b \Rightarrow a = u/2 - b = \mathbf{36.6\,m}$
    - $d = \sqrt{a^2+b^2} = \sqrt{36.6^2+24.3^2} = \sqrt{1930.05} \approx \mathbf{43.93\,m}$

- $A = a \cdot b = 36.6 \cdot 24.3 = \mathbf{889.38\,m^2}$

(c)
- $b = \sqrt{d^2 - a^2} = \sqrt{15^2 + 8^2} = \sqrt{161} \approx \mathbf{12.69\,cm}$
- $u = 2(a + b) = 2(8 + \sqrt{161}) \approx \mathbf{41.38\,cm}$
- $A = a \cdot b = 8 \cdot \sqrt{161} \approx \mathbf{101.51\,cm^2}$

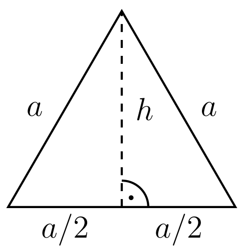
12. (a) $\sqrt{13^2 - 5^2} = \sqrt{169 - 25} = \sqrt{144} = \mathbf{12\,mm}$
    (b) $\sqrt{6^2 + 10^2} = \sqrt{36 + 100} = \sqrt{136} \approx \mathbf{11.66\,m}$
    (c) $\sqrt{12^2 - 6^2} = \sqrt{144 - 36} = \sqrt{108} \approx \mathbf{10.39\,cm}$
    oder mit der Formel für die Höhe im gleichseitigen Dreieck:
    $\sqrt{3} \cdot \frac{a}{2} = \sqrt{3} \cdot 6 = \mathbf{10.39\,cm}$

13. Planfigur:



- $c/2 = \sqrt{a^2 - h^2} \quad \Rightarrow \quad c = 2\sqrt{65^2 - 52^2} = 2\sqrt{1521} = \mathbf{78\,cm}$
- $A = \frac{c \cdot h_c}{2} = \frac{78 \cdot 52}{2} = \mathbf{2028\,mm^2}$

14. Planfigur:

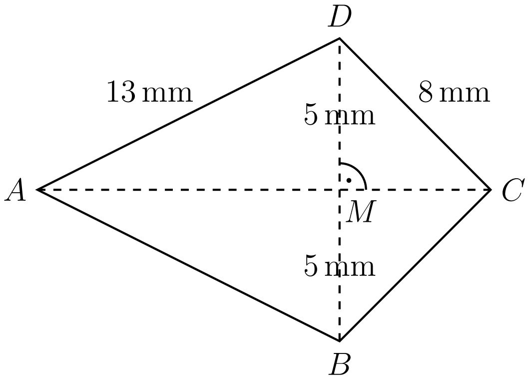


Direkt mit Formel (siehe Theorie):

$h = \sqrt{3} \cdot \frac{a}{2} = \sqrt{3} \cdot 5 = \mathbf{8.66\,cm}$

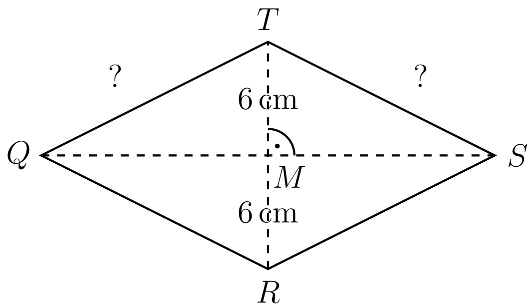
$A = \frac{\sqrt{3} \cdot a^2}{4} = \frac{\sqrt{3} \cdot 10^2}{4} = \mathbf{43.30\,cm^2}$

15. (a) Planfigur:



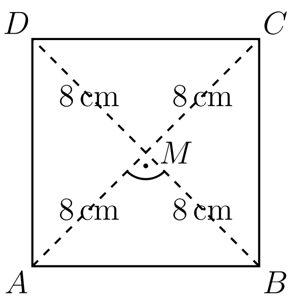
- $|AM| = \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = \sqrt{144} = \mathbf{12\,mm}$
- $|MC| = \sqrt{8^2 - 5^2} = \sqrt{64 - 25} = \sqrt{39} \approx \mathbf{6.24\,mm}$
- $|AC| = |AM| + |MC| \approx \mathbf{18.24\,mm}$

(b) Planfigur: ($|QS| = 40\,\text{cm}$)



- $|QM| = 40 : 2 = 20\,\text{cm}$
- $|QR| = |RS| = |ST| = |TQ| = \sqrt{20^2 + 6^2} = \sqrt{436} \approx \mathbf{20.88\,cm}$
- $A = |QS| \cdot |RT| : 2 = 40 \cdot 12 : 2 = \mathbf{240\,cm^2}$

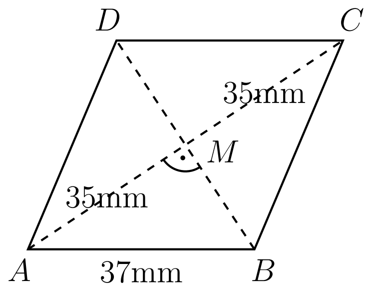
16. Planfigur:



- $a = \sqrt{8^2 + 8^2} = \sqrt{128} \approx 11.31\,\text{cm}$
- $A = \sqrt{128}^2 = 128\,\text{cm}^2$

17. Gegeben: Rhombus mit $a = 37\,\text{mm}$ und $|AC| = 70\,\text{mm}$

    Gesucht: $|BD| =?$



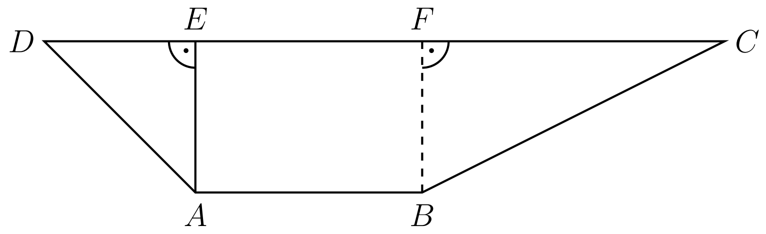
- $|BM| = \sqrt{37^2 - 35^2} = 12\,\text{mm} \quad \Rightarrow \quad |BD| = 2 \cdot |BM| = \mathbf{24\,mm}$
- $A = |AC| \cdot |BD| : 2 = 70 \cdot 24 : 2 = 70 \cdot 12 = \mathbf{840\,mm^2}$

18. $A = 144\,\text{cm}^2 \quad \Rightarrow \quad a = \sqrt{A} = 12\,\text{cm} \quad \Rightarrow \quad d = \sqrt{2} \cdot a = \sqrt{2} \cdot 12 \approx \mathbf{16.97\,cm}$

19. (a) Wegen $\alpha = 45°$ ist das rechtwinklige Dreieck, das durch die Seite AC und die Höhe gebildet wird, auch *gleichschenklig*.

    Folglich ist AD die Diagonale eines Quadrats mit der Seitenlänge 50 mm und es gilt: $|AD| = |BC| = \sqrt{2} \cdot 50 \approx \mathbf{70.17\,mm}$

20. (a) Planfigur:



- $|DE| = \sqrt{20^2 - 16^2} = 12\,\text{cm}$
- $|FC| = \sqrt{34^2 - 16^2} = 30\,\text{cm}$
- $|DC| = |DE| + |EF| + |FC| = 12\,\text{cm} + 16\,\text{cm} + 30\,\text{cm} = \mathbf{58\,cm}$

21. (a)
- Pythagoras: $c = \sqrt{a^2 + b^2} = \sqrt{45^2 + 60^2} = \sqrt{5625} = \mathbf{75\,cm}$
- Kathetensatz: $a^2 = p \cdot c \quad \Rightarrow \quad p = a^2 : c = 45^2 : 75 = \mathbf{27\,cm}$
- Hypotenusenabschnitte: $q = c - p = 75 - 27 = \mathbf{48\,cm}$
- Höhensatz: $h^2 = p \cdot q \quad \Rightarrow \quad h = \sqrt{p \cdot q} = \sqrt{27 \cdot 48} = \sqrt{1296} = \mathbf{36\,cm}$
- $A = a \cdot b : 2 = 45 \cdot 60 : 2 = \mathbf{1350\,cm^2}$

(b)
- $A = a \cdot b : 2 \quad \Rightarrow \quad a = 2A : b = 300 : 15 = \mathbf{20\,dm^2}$
- Pythagoras: $c = \sqrt{a^2 + b^2} = \sqrt{20^2 + 15^2} = \sqrt{625} = \mathbf{25\,dm}$
- Kathetensatz: $a^2 = p \cdot c \quad \Rightarrow \quad p = a^2 : c = 20^2 : 25 = \mathbf{8\,dm}$
- $q = c - p = 25 - 8 = \mathbf{16\,dm}$
- Höhensatz: $h^2 = p \cdot q \quad \Rightarrow \quad h = \sqrt{p \cdot q} = \sqrt{8 \cdot 16} = \sqrt{144} = \mathbf{12\,dm}$

(c) *Hinweis:* eine der drei Angaben ist überflüssig und könnte weggelassen werden (z. B. $h = 60\,\text{m}$).

- Kathetensatz: $a^2 = p \cdot c \quad \Rightarrow \quad a = \sqrt{p \cdot c} = \sqrt{144 \cdot 169} = \mathbf{156\,m}$
- Pythagoras: $b = \sqrt{c^2 - a^2} \cdot b : 2 = \sqrt{169^2 - 156^2} = \sqrt{4225} = \mathbf{65\,m}$
- Hypotenusenabschnitte: $q = c - p = 169 - 144 = \mathbf{25\,m}$
- $A = a \cdot b : 2 = 156 \cdot 65 : 2 = \mathbf{5070\,m^2}$

22. (a)
- $u = 2a + c = 2 \cdot 52 + 40 = \mathbf{144\,cm}$
- $h = \sqrt{a^2 - (c/2)^2} = \sqrt{52^2 - 20^2} = \sqrt{2304} = \mathbf{48\,cm}$
- $A = c \cdot h_c : 2 = 40 \cdot 48 : 2 = \mathbf{960\,cm^2}$

(b)
- $u = 2a + c \quad \Rightarrow \quad a = (u - c)/2 = (28 - 8)/2 = \mathbf{10\,dm}$
- $h = \sqrt{a^2 - (c/2)^2} = \sqrt{10^2 - 4^2} = \sqrt{84} = \mathbf{9.17\,dm}$
- $A = c \cdot h_c/2 = 8 \cdot 9.17/2 = \mathbf{36.66\,dm}^2$

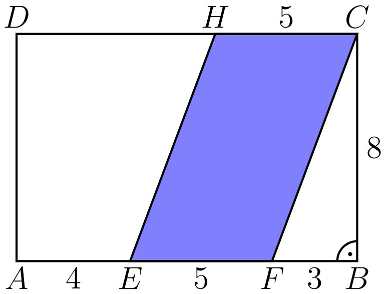
(c)
- $A = c \cdot h_c/2 \quad \Rightarrow \quad c = 2A/h_c = 300/15 = \mathbf{20\,cm}$
- $a = \sqrt{h^2 + (c/2)^2} = \sqrt{15^2 + 10^2} = \sqrt{325} \approx \mathbf{18.03\,cm}$
- $u = 2a + c = \mathbf{56.06\,cm}$

23. (a)
- $e/2 = \sqrt{s^2 - (f/2)^2} \quad \Rightarrow \quad e = 2\sqrt{s^2 - (f/2)^2} = 2\sqrt{37^2 - 12^2} = \mathbf{70\,cm}$
- $u = 4s = 4 \cdot 37\,\text{cm} = \mathbf{148\,cm}$
- $A = e \cdot f/2 = 70 \cdot 24/2 = \mathbf{84\,cm}^2$

(b)
- $s = \sqrt{(e/2)^2 + (f/2)^2} = \sqrt{7.2^2 + 6^2} \approx \mathbf{9.37\,m}$
- $u = 4s = 4 \cdot 9.37\,\text{cm} = \mathbf{37.49\,m}$
- $A = e \cdot f/2 = 14.4 \cdot 12/2 = \mathbf{86.4\,m}^2$

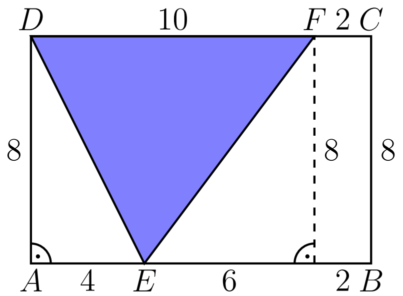
(c)
- $u = 4s \quad \Rightarrow \quad s = u/4 = 64/4 = \mathbf{16\,cm}$
- $f/2 = \sqrt{s^2 - (e/2)^2} \quad \Rightarrow \quad f = 2\sqrt{s^2 - (e/2)^2} = 2\sqrt{16^2 - 6^2} \approx \mathbf{29.66\,cm}$
- $A = e \cdot f/2 = 12 \cdot 29.66/2 = \mathbf{177.99\,cm}^2$

32. (a) Planfigur: (alle Angaben in cm)



- $|EH| = |FC| = \sqrt{8^2 + 3^2} = \sqrt{73} \approx 8.54\,\text{cm}$
- $u = 2(|EF| + |FC|) \approx \mathbf{27.09\,cm}$

(b) Planfigur: (alle Angaben in cm)



- $|DE| = \sqrt{8^2 + 4^2} = \sqrt{80}\,\text{cm}$
- $|EF| = \sqrt{8^2 + 6^2} = 10\,\text{cm}$
- $u = |DE| + |EF| + |FD| = \sqrt{80} + 10 + 10 \approx \mathbf{28.94\,cm}$

43. (a) Wenn man weiss, dass die Länge einer Strecke $|PQ|$ mit den Endpunkten (x_P, y_P) und (x_Q, y_Q) nach der Formel

$$|PQ| = \sqrt{(x_Q - x_P)^2 + (y_Q - y_P)^2}$$

berechnet wird, muss man das Trapez nicht im Koordinatensystem darstellen. Wegen der Quadrate spielt es übrigens keine Rolle, ob man die Differenzen $x_Q - x_P$ oder $x_P - x_Q$ bzw. $y_Q - y_P$ oder $y_P - y_Q$ berechnet.

- Umfang:
  $|AB| = \sqrt{9^2 + 0^2} = 9$
  $|BC| = \sqrt{2^2 + 4^2} = \sqrt{20}$
  $|CD| = \sqrt{3^2 + 0^2} = 3$
  $|DA| = \sqrt{4^2 + 4^2} = \sqrt{32}$
  $u = 9 + \sqrt{20} + 3 + \sqrt{32} = \mathbf{22.13\,LE}$ (Längeneinheiten)
- Flächeninhalt (hier mit F bezeichnet) mit der Dreiecksmethode:
  $$2F_{A,B} = \begin{vmatrix} 1 & 0 \\ 10 & 0 \end{vmatrix} = 1 \cdot 0 - 0 \cdot 10 = 0$$
  $$2F_{B,C} = \begin{vmatrix} 10 & 0 \\ 8 & 4 \end{vmatrix} = 10 \cdot 4 - 0 \cdot 8 = 40$$
  $$2F_{C,D} = \begin{vmatrix} 8 & 4 \\ 5 & 4 \end{vmatrix} = 8 \cdot 4 - 5 \cdot 4 = 12$$
  $$2F_{D,A} = \begin{vmatrix} 5 & 4 \\ 1 & 0 \end{vmatrix} = 5 \cdot 0 - 4 \cdot 1 = -4$$
  $2F = 0 + 40 + 12 + (-4) = 48 \quad \Rightarrow \quad F = \mathbf{24\,FE}$ (Flächeneinheiten)

46. (a)
- $|AB| = \sqrt{3^2 + 4^2} = \mathbf{5}$
- $|CD| = \sqrt{3^2 + 4^2} = \mathbf{5}$
- $|EF| = \sqrt{4^2 + 5^2} = \mathbf{\sqrt{29}}$
- $|GH| = \sqrt{3^2 + 0^2} = \mathbf{3}$
- $|IJ| = \sqrt{0^2 + 3^2} = \mathbf{3}$
- $|KL| = \sqrt{2^2 + 5^2} = \mathbf{\sqrt{29}}$
- $|MN| = \sqrt{4.5^2 + 6^2} = \mathbf{7.5}$

Satz des Pythagoras **Lösungen+** **Buch (F3)**

25. (a) $k = \sqrt{10^2 + 7^2 + 4.5^2} = \sqrt{169.25} = \sqrt{169.25} \approx \mathbf{13.01\,cm}$
(b) $k = \sqrt{6^2 + 9^2 + 5.5^2} = \sqrt{147.25} \approx \mathbf{12.13\,mm}$
(c) $k = \sqrt{4.5^2 + 7^2 + 10^2} = \sqrt{169.25} \approx \mathbf{13.01\,cm}$
(d) $k = \sqrt{8^2 + 9.4^2 + 5.5^2} = \sqrt{182.61} \approx \mathbf{13.51\,dm}$
(e) $k = \sqrt{10^2 + 3.4^2 + 6.8^2} = \sqrt{157.8} \approx \mathbf{12.56\,cm}$
(f) $k = \sqrt{7^2 + 7^2 + 7^2} = \sqrt{147} \approx \mathbf{12.12\,dm}$
(g)
- $k = \mathbf{\sqrt{a^2 + b^2 + c^2}}$
- $k = \sqrt{a^2 + a^2 + a^2} = \sqrt{3a^2} = \sqrt{3} \cdot \sqrt{a^2} = \mathbf{\sqrt{3} \cdot a}$