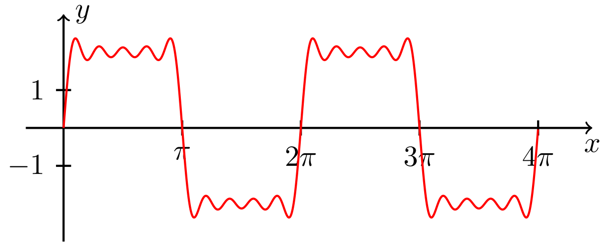
Aufgabe 1

$y = \frac{8}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \frac{1}{7}\sin 7x + \frac{1}{9}\sin 9x + \dots\right)$

Die Funktion, welche durch diese Fourierreihe dargestellt wird, ist eine „Rechteckfunktion“

$$f(x) = \begin{cases} -2 & \text{für } -\pi < x < 0 \\ 2 & \text{für } 0 < x < \pi \end{cases} \quad \text{mit} \quad f(x + 2\pi) = f(x)$$



Aufgabe 2

(k) ungerade

(l) gerade, denn $f(-x) = \sin([-x]^2) = \sin(x^2) = f(x)$

(m) gerade, denn $f(-x) = (-x^5) \cdot \sin(-x) = -x^5 \cdot (-\sin x) = x^5 \cdot \sin x = f(x)$

(n) gerade, denn $f(-x) = 3 = f(x)$

(o) ungerade, denn $f(-x) = \sin(-4x) \cdot \cos(-5x) = -\sin(4x) \cdot \cos(5x) = -f(x)$

(p) gerade und ungerade, denn $f(-x) = 0 = f(x)$ und $f(-x) = 0 = -0 = -f(x)$

(q) gerade, da Produkt aus zwei ungeraden Funktionen

(r) gerade

(s) weder gerade noch ungerade, da gerade *und* ungerade Exponenten vorkommen.

(t) gerade

Aufgabe 3

Bestimme die Periode bzw. Schwingungsdauer T der folgenden Funktionen. (Winkel im Bogenmass)

(a) $f(x) = \sin x$

$\sin(x) = \sin(x + 2\pi)$

$f(x) = f(x + 2\pi) \quad \Rightarrow \quad T = 2\pi$

(b) $f(t) = \tan t$

$\tan(t) = \tan(t + \pi)$

$f(t) = f(t + \pi) \quad \Rightarrow \quad T = \pi$

($T' = 2\pi$ wäre nicht minimal)

(c) $g(x) = \cos\left(x + \frac{\pi}{2}\right)$

$\cos(x + \frac{\pi}{2}) = \cos(x + \frac{\pi}{2} + 2\pi)$

$g(x) = g(x + 2\pi) \quad \Rightarrow \quad T = 2\pi$

(d) $h(x) = \begin{cases} 0 & \text{für } -1 < x \leq 0 \\ 1 & \text{für } 0 < x \leq 1 \\ & h(x+2) = h(x) \end{cases}$

Aus $h(x + 2) = h(x)$ folgt unmittelbar $T = 2$

(e) $f(t) = \sin(4t)$

$\sin(4t) = \sin(4t + 2\pi) = \sin\left(4\left[t + \frac{\pi}{2}\right]\right)$

$f(t) = f\left(t + \frac{\pi}{2}\right) \quad \Rightarrow \quad T = \frac{\pi}{2}$

(f) $g(t) = \cos(3\pi t)$

$\cos(3\pi t) = \cos(3\pi t + 2\pi) = \cos\left(3\pi\left[t + \frac{2}{3}\right]\right)$

$g(t) = g(t + \frac{2}{3}) \quad \Rightarrow \quad T = \frac{2}{3}$

(g) $h(x) = \cos(\frac{7}{\pi}x)$

$\cos\left(\frac{7}{\pi}x\right) = \cos\left(\frac{7}{\pi}x + 2\pi\right) = \cos\left(\frac{7}{\pi}\left[x + \frac{2}{7}\pi^2\right]\right)$

$h(x) = h(x + \frac{2}{7}\pi^2) \quad \Rightarrow \quad T = \frac{2}{7}\pi^2$

(h) $f(x) = \cos(2x + \pi)$

$\cos(2x + \pi) = \cos(2x + \pi + 2\pi) = \cos\left(2\left[x + \pi\right] + \pi\right)$

$f(x) = f(x + \pi) \quad \Rightarrow \quad T = \pi$

(i) $f(x) = \cos\left(2(x + \pi)\right)$

$\cos\left(2(x + \pi)\right) = \cos\left(2x + 2\pi + 2\pi\right) = \cos\left(2[x + \pi] + 2\pi\right)$

$f(x) = f(x + \pi) \quad \Rightarrow \quad T = \pi$

Aufgabe 4

(a) $\cos(3x) \cdot \cos(6x) = \frac{1}{2}\left[\cos(9x) + \cos(-3x)\right]$

$= \frac{1}{2}\left[\cos(9x) + \cos(3x)\right]$

(b) $\sin(4t) \cdot \cos(3t) = \frac{1}{2}\left[\sin t + \sin(7t)\right]$

(c) $\sin(5x) \cdot \sin(2x) = \frac{1}{2}\left[\cos(3x) - \cos(7x)\right]$

(d) $\cos(2x) \cdot \cos(2x) = \frac{1}{2}\left[\cos 0 + \cos(4x)\right]$

$= \frac{1}{2}\left[1 + \cos(4x)\right]$

Aufgabe 5

(a) $\int_{-2}^{2} (x^3 - x)\,\mathrm{d}x = 0$

ungerader Integrand, symmetrische Grenzen

(b) $\int_{25\pi}^{27\pi} \cos x\,\mathrm{d}x = 0$

Cosinuskurve über $T = 2\pi$ integriert

(c) $\int_{-\pi/2}^{\pi/2} \cos x \cdot \sin x\,\mathrm{d}x = 0$

ungerader Integrand, symmetrische Grenzen

(d) $\int_{0}^{2\pi} \sin(x+3)\,\mathrm{d}x = 0$

verschobene Sinuskurve, über $T = 2\pi$ integriert

Aufgabe 6

(a) $\int_{0}^{2\pi} x \cdot \sin x\,\mathrm{d}x = \dots$

$f'(x) = \sin x \quad \Rightarrow \quad f(x) = -\cos x$

$g(x) = x \quad \Rightarrow \quad g'(x) = 1$

$$\ldots = \left[-\cos x \cdot x \right]_0^{2\pi} - \int_0^{2\pi} 1 \cdot (-\cos x) \, \mathrm{d}x$$

$$= 2\pi - 0$$

$$= 2\pi$$

(b) $\int_0^{2\pi} x^2 \cdot \cos x \, \mathrm{d}x = \ldots$

$$f'(x) = \cos x \quad \Rightarrow \quad f(x) = \sin x$$
$$g(x) = x^2 \quad \Rightarrow \quad g'(x) = 2x$$

$$\ldots = \left[\sin x \cdot x^2 \right]_0^{2\pi} - \int_0^{2\pi} 2x \cdot \sin x \, \mathrm{d}x$$

$$= 0 - 2 \int_0^{2\pi} x \cdot \sin x \, \mathrm{d}x$$

$$\overset{\text{(a)}}{=} -2 \cdot 2\pi = -4\pi$$

Aufgabe 7

(a) $a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} x^3 \, \mathrm{d}x = 0$

$$a_1 = \frac{1}{\pi} \int_{-\pi}^{\pi} x^3 \cos x \, \mathrm{d}x = 0$$

$$a_2 = \frac{1}{\pi} \int_{-\pi}^{\pi} x^3 \cos 2x \, \mathrm{d}x = 0$$

$$b_1 = \frac{1}{\pi} \int_{-\pi}^{\pi} x^3 \sin x \, \mathrm{d}x = 7.7392$$

$$b_2 = \frac{1}{\pi} \int_{-\pi}^{\pi} x^3 \sin 2x \, \mathrm{d}x = -8.3696$$

$$b_3 = \frac{1}{\pi} \int_{-\pi}^{\pi} x^3 \sin 3x \, \mathrm{d}x = 6.1353$$

$$b_4 = \frac{1}{\pi} \int_{-\pi}^{\pi} x^3 \sin 4x \, \mathrm{d}x = -4.7473$$

(b) $a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} \mathrm{e}^{-x^2} \, \mathrm{d}x = 0.2821$

$$a_1 = \frac{1}{\pi} \int_{-\pi}^{\pi} \mathrm{e}^{-x^2} \cos x \, \mathrm{d}x = 0.4394$$

$$a_2 = \frac{1}{\pi} \int_{-\pi}^{\pi} \mathrm{e}^{-x^2} \cos(2x) \, \mathrm{d}x = 0.2075$$

$$a_3 = \frac{1}{\pi} \int_{-\pi}^{\pi} \mathrm{e}^{-x^2} \cos(3x) \, \mathrm{d}x = 0.0595$$

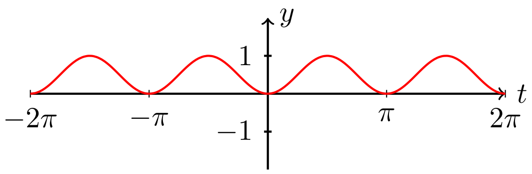
$$b_1 = \frac{1}{\pi} \int_{-\pi}^{\pi} \mathrm{e}^{-x^2} \sin x \,\mathrm{d}x = 0$$

$$b_2 = \frac{1}{\pi} \int_{-\pi}^{\pi} \mathrm{e}^{-x^2} \sin(2x) \,\mathrm{d}x = 0$$

$$b_3 = \frac{1}{\pi} \int_{-\pi}^{\pi} \mathrm{e}^{-x^2} \sin(3x) \,\mathrm{d}x = 0$$

Aufgabe 8

(a) Graph:



$f(x) = x^2$ ist eine gerade Funktion

$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} \sin^2 x \,\mathrm{d}x = \frac{1}{2}$$

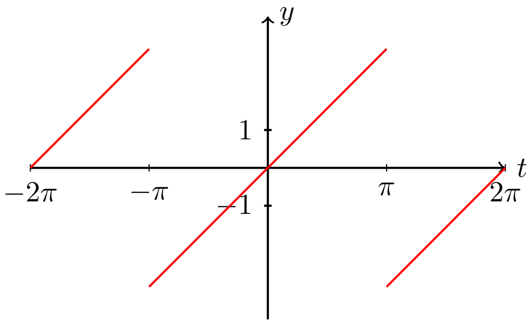
$$a_1 = \frac{1}{\pi} \int_0^{2\pi} \sin^2 x \cdot \cos x \,\mathrm{d}x = 0$$

$$a_2 = \frac{1}{\pi} \int_0^{2\pi} \sin^2 x \cdot \cos 2x \,\mathrm{d}x = -\frac{1}{2}$$

$$a_3 = \frac{1}{\pi} \int_0^{2\pi} \sin^2 x \cdot \cos 3x \,\mathrm{d}x = 0 \quad \text{(usw.)}$$

$$f(x) = \frac{1}{2} - \frac{1}{2} \cos 2x \qquad \text{(endliche Fourierreihe!)}$$

(b) Graph:



$f(x) = x$ ist eine ungerade Funktion $\Rightarrow$ nur b_n-Terme.

$$b_1 = \frac{1}{\pi}\int_{-\pi}^{\pi} x \cdot \sin x \, \mathrm{d}x = 2 = \frac{2}{1}$$

$$b_2 = \frac{1}{\pi}\int_{-\pi}^{\pi} x \cdot \sin 2x \, \mathrm{d}x = -1 = -\frac{2}{2}$$

$$b_3 = \frac{1}{\pi}\int_{-\pi}^{\pi} x \cdot \sin 3x \, \mathrm{d}x = \frac{2}{3}$$

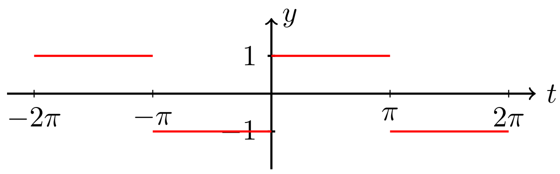
$$b_4 = \frac{1}{\pi}\int_{-\pi}^{\pi} x \cdot \sin 4x \, \mathrm{d}x = -\frac{1}{2} = -\frac{2}{4}$$

$$b_5 = \frac{1}{\pi}\int_{-\pi}^{\pi} x \cdot \sin 5x \, \mathrm{d}x = \frac{2}{5}$$

$$f(x) = \sum_{k=1}^{\infty}(-1)^{k+1} \cdot \frac{2}{k} \cdot \sin(kx)$$

Aufgabe 9

(a) Graph:



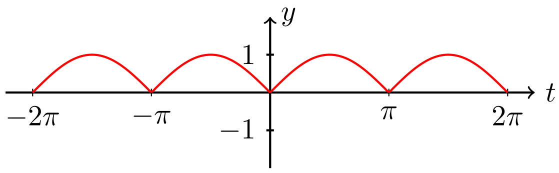
f ist ungerade $\Rightarrow$ nur b_n-Terme.

$$\begin{aligned}
b_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,\mathrm{d}x \\
&= \frac{1}{\pi}\int_{-\pi}^{0}(-1)\sin(nx)\,\mathrm{d}x + \frac{1}{\pi}\int_{0}^{\pi}\sin(nx)\,\mathrm{d}x \\
&= \left[\frac{1}{n\pi}\cos(nx)\right]_{-\pi}^{0} + \left[\frac{-1}{n\pi}\cos(nx)\right]_{0}^{\pi} \\
&\overset{(*)}{=} \frac{1}{n\pi}\cos(0) - \frac{1}{n\pi}\cos(n\pi) - \frac{1}{n\pi}\cos(n\pi) + \frac{1}{n\pi}\cos(0) \\
&= \frac{2}{n\pi} - \frac{2}{n\pi}(-1)^n \qquad {\scriptstyle (*)\ \cos(-x) = \cos(x)}
\end{aligned}$$

$$b_n = \begin{cases} 0 & \text{falls } n \text{ gerade} \\ \dfrac{4}{n\pi} & \text{falls } n \text{ ungerade} \end{cases}$$

$$f(x) = \frac{4}{\pi}\sin x + \frac{4}{3\pi}\sin 3x + \frac{4}{5\pi}\sin 5x + \dots$$

(b) Graph:



f ist gerade $\Rightarrow$ nur a_n-Terme.

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |\sin x|\,\mathrm{d}x = 2\cdot\frac{1}{2\pi}\int_0^{\pi}\sin x\,\mathrm{d}x$$

$$= \frac{1}{\pi}\left[-\cos x\right]_0^{\pi} = \frac{1}{\pi}\left(-\cos\pi+\cos 0\right) = \frac{1}{\pi}\cdot 2 = \frac{2}{\pi}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} |\sin x|\cos(nx)\,\mathrm{d}x$$

$$= 2\cdot\frac{1}{\pi}\int_0^{\pi}\sin x\cos(nx)\,\mathrm{d}x \quad [\to \text{FBT. S. 99}]$$

$$= \frac{2}{\pi}\int_0^{\pi}\frac{1}{2}\left[\sin[(1+n)x]+\sin[(1-n)x]\right]\,\mathrm{d}x$$

$$= \frac{1}{\pi}\left[\frac{-1}{1+n}\cos[(1+n)x] - \frac{1}{1-n}\cos[(1-n)x]\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{-1}{1+n}\cos[(1+n)\pi] - \frac{1}{1-n}\cos[(1-n)\pi] + \frac{1}{1+n} + \frac{1}{1-n}\right]$$

$$= \frac{1}{\pi}\left[\frac{(-1)^n}{1+n} + \frac{(-1)^n}{1-n} + \frac{1}{1+n} + \frac{1}{1-n}\right]$$

- Falls n gerade:

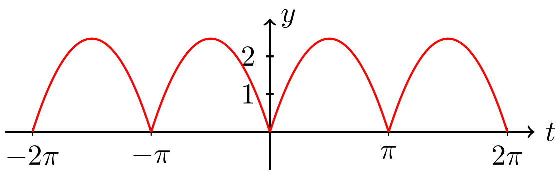
  $$a_n = \frac{1}{\pi}\left[\frac{2}{1+n}+\frac{2}{1-n}\right] = \cdots = \frac{4}{\pi(1-n^2)}$$

- Falls n ungerade:

  $$a_n = \frac{1}{\pi}\cdot 0 = 0$$

$$f(x) = \frac{2}{\pi} - \frac{4}{3\pi}\cos 2x - \frac{4}{15\pi}\cos 4x - \frac{4}{35\pi}\cos 6x - \ldots$$

(c) Graph:



f ist gerade $\Rightarrow$ nur a_n-Terme.

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,\mathrm{d}x = 2\cdot\frac{1}{2\pi}\int_0^{\pi}(\pi x - x^2)\,\mathrm{d}x$$

$$= \frac{1}{\pi}\left[\frac{\pi}{2}x^2 - \frac{1}{3}x^3\right]_0^{\pi} = \frac{1}{\pi}\left(\frac{\pi^3}{2} - \frac{\pi^3}{3}\right)$$

$$= \frac{\pi^2}{6}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,\mathrm{d}x$$

$$= 2\cdot\frac{1}{\pi}\int_0^{\pi}(\pi x - x^2)\cos(nx)\,\mathrm{d}x = \dots$$

$$u'(x) = \cos(nx) \quad \Rightarrow \quad u(x) = \frac{1}{n}\sin(nx)$$

$$v(x) = \pi x - x^2 \quad \Rightarrow \quad v'(x) = \pi - 2x$$

$$\dots = \frac{2}{\pi}\left\{\underbrace{\left[(\pi x - x^2)\frac{1}{n}\sin(nx)\right]_0^{\pi}}_{0} - \int_0^{\pi}(\pi - 2x)\frac{1}{n}\sin(nx)\,\mathrm{d}x\right\}$$

$$= \frac{2}{\pi n}\cdot\int_0^{\pi}(2x - \pi)\sin(nx)\,\mathrm{d}x$$

$$a_n = \frac{2}{\pi n}\cdot\int_0^{\pi}(2x - \pi)\sin(nx)\,\mathrm{d}x \quad = \dots$$

$$\boxed{\begin{aligned} u'(x) &= \sin(nx) \quad \Rightarrow \quad u(x) = -\frac{1}{n}\cos(nx) \\ v(x) &= 2x - \pi \quad \Rightarrow \quad v'(x) = 2 \end{aligned}}$$

$$\dots = \frac{2}{\pi n}\cdot\left\{\left[(2x - \pi)\frac{-1}{n}\cos(nx)\right]_0^{\pi} - \int_0^{\pi} 2\frac{-1}{n}\cos(nx)\,\mathrm{d}x\right\}$$

$$= \frac{2}{\pi n^2}\left\{\big[(\pi - 2x)\cos(nx)\big]_0^{\pi} + 2\underbrace{\int_0^{\pi}\cos(nx)\,\mathrm{d}x}_{0}\right\}$$

$$= \frac{2}{\pi n^2}\big[(\pi - 2\pi)\cos(n\pi) - (\pi - 0)\big]$$

$$a_n = \frac{2}{\pi n^2}\big[-\pi\cos(n\pi) - \pi\big]$$

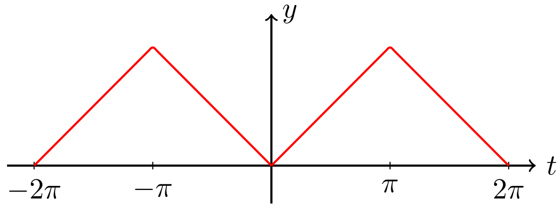
$$= \frac{2}{n^2}\big[-\cos(n\pi) - 1\big]$$

$$= \frac{2}{n^2}\big[(-1)^{n+1} - 1\big]$$

$$a_n = \begin{cases} 0 & \text{falls } n \text{ gerade} \\ -\dfrac{4}{n^2} & \text{falls } n \text{ ungerade} \end{cases}$$

$$f(x) = \frac{\pi^2}{6} - \frac{4}{4}\cos 2x - \frac{4}{16}\cos 4x - \frac{4}{36}\cos 6x - \dots$$

(d) Graph:



f ist gerade ⇒ nur a_n-Terme.

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |x|\,\mathrm{d}x = 2\cdot\frac{1}{2\pi}\int_0^{\pi} x\,\mathrm{d}x$$
$$= \frac{1}{\pi}\left[\frac{1}{2}x^2\right]_0^{\pi} = \frac{1}{\pi}\cdot\frac{1}{2}\pi^2 = \frac{\pi}{2}$$

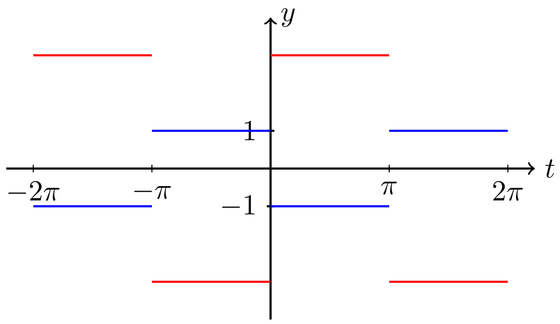
$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} |x|\cos(nx)\,\mathrm{d}x = \frac{2}{\pi}\int_0^{\pi} x\cos(nx)\,\mathrm{d}x$$
$$= \frac{2}{\pi}\left[x\cdot\frac{1}{n}\sin(nx)\right]_0^{\pi} - \frac{2}{\pi}\int_0^{\pi} 1\cdot\frac{1}{n}\sin(nx)\,\mathrm{d}x$$
$$= \frac{2}{n\pi}\left[x\cdot\sin(nx)\right]_0^{\pi} - \frac{2}{n\pi}\int_0^{\pi}\sin(nx)\,\mathrm{d}x$$
$$= \frac{2}{n\pi}\cdot\pi\cdot\underbrace{\sin(n\pi)}_{0} - \frac{2}{n\pi}\left[\frac{-1}{n}\cos(nx)\right]_0^{\pi}$$
$$= \frac{2}{n^2\pi}\left[\cos(nx)\right]_0^{\pi} = \frac{2}{n^2\pi}\left[\cos(n\pi)-1\right] = \frac{2}{n^2\pi}\left[(-1)^n - 1\right]$$

$$a_n = \begin{cases} 0 & \text{falls } n \text{ gerade} \\ -\dfrac{4}{n^2\pi} & \text{falls } n \text{ ungerade} \end{cases}$$

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\cos x - \frac{4}{9\pi}\cos 3x - \frac{4}{25\pi}\cos 5x - \dots$$

Aufgabe 10

(a) Graph:



$f(x) = (-3) \cdot g(x)$

Die Fourierreihe von

$$g(x) = \begin{cases} -1 & \text{für } -\pi < x < 0 \\ 1 & \text{für } 0 < x < \pi \end{cases}$$

wurde in Aufgabe 9a bestimmt

$$f(x) = (-3) \cdot \left[\frac{4}{\pi} \sin x + \frac{4}{3\pi} \sin(3x) + \frac{4}{5\pi} \sin(5x) + \dots\right]$$

$$f(x) = -\frac{12}{\pi} \sin x + \frac{12}{3\pi} \sin(3x) + \frac{12}{5\pi} \sin(5x) + \dots$$

Gegeben: Fourierreihe von $g(x) = \begin{cases} -1 & \text{für } -\pi < x < 0 \\ 1 & \text{für } 0 < x < \pi \end{cases}$

$$g(x) = \frac{4}{\pi} \sin x - \frac{4}{3\pi} \sin 3x - \frac{4}{5\pi} \sin 5x - \dots$$

Gesucht: Fourierreihe von $f(x) = \begin{cases} 3 & \text{für } -\pi < x < 0 \\ -3 & \text{für } 0 < x < \pi \end{cases}$

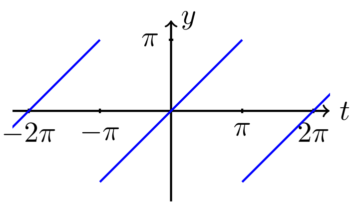
Offenbar ist:

$f(x) = (-3) \cdot g(x)$

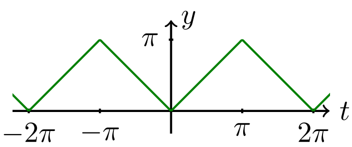
$$f(x) = (-3) \cdot \left[\frac{4}{\pi} \sin x - \frac{4}{3\pi} \sin 3x - \frac{4}{5\pi} \sin 5x - \dots\right]$$

$$f(x) = -\frac{12}{\pi} \sin x + \frac{12}{3\pi} \sin 3x + \frac{12}{5\pi} \sin 5x + \dots$$

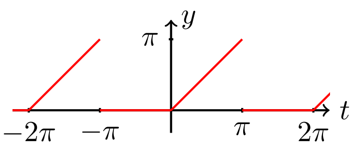
(b) Graph:



$y = g(x)$



$y = h(x)$



$y = \frac{1}{2}\big(g(x) + h(x)\big)$

$$f(x) = \frac{1}{2} \cdot \big(x + |x|\big)$$

Die Fourierreihe von $g(x) = x$ wurde in Aufgabe 8b bestimmt

Die Fourierreihe von $h(x) = |x|$ wurde in Aufgabe 9d bestimmt

$$f(x) = \frac{1}{2} \cdot \left[2 \sin x - \frac{2}{2} \sin(2x) + \frac{2}{3} \sin(3x) - \frac{2}{4} \sin(4x) + \dots\right] + \frac{1}{2}\left[\frac{\pi}{2} - \frac{4}{\pi} \cos x - \frac{4}{9\pi} \cos(3x) - \frac{4}{25\pi} \cos(5x) - \dots\right]$$

Das „vereinfachen“ dieser Summe ersparen wir uns ausnahmsweise.

Gegeben: Fourierreihen

$$g(x) = x = 2\sum_{k=1}^{\infty} \frac{1}{k}\sin(kx) \quad \text{für } -\pi < x < \pi$$

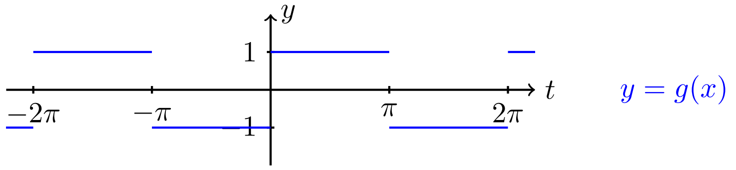
$$h(x) = |x| = \frac{\pi}{2} - 4\sum_{k=1}^{\infty} \frac{1}{k^2}\cos(kx) \quad \text{für } -\pi < x < \pi$$

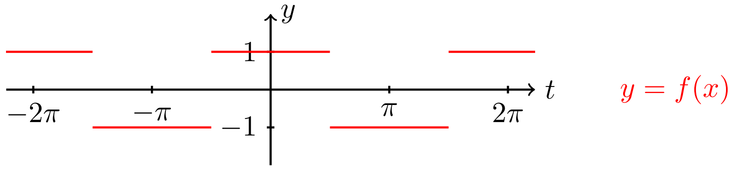
Gesucht: Fourierreihe von $f(x) = \begin{cases} 0 & \text{für } -\pi < x < 0 \\ x & \text{für } 0 < x < \pi \end{cases}$

Trick: $f(x) = \frac{1}{2}\left[g(x) + h(x)\right]$

$$\begin{aligned} f(x) &= \frac{1}{2}\left[g(x) + h(x)\right] \\ &= \frac{1}{2}\left[2\sum_{k=1}^{\infty}\frac{1}{k}\sin kx + \frac{\pi}{2} - 4\sum_{k=1}^{\infty}\frac{1}{k^2}\cos kx\right] \\ &= \frac{\pi}{4} - 2\sum_{k=1}^{\infty}\frac{1}{k^2}\cos kx + \sum_{k=1}^{\infty}\frac{1}{k}\sin kx \end{aligned}$$

(c) Graph





$f(x) = g\left(x + \frac{\pi}{2}\right)$

Die Fourierreihe von

$$g(x) = \begin{cases} -1 & \text{für } -\pi < x < 0 \\ 1 & \text{für } 0 < x < \pi \end{cases}$$

wurde in Aufgabe 9a bestimmt

$$f(x) = \frac{4}{\pi}\sin\left(x + \frac{\pi}{2}\right) + \frac{4}{3\pi}\sin\left[3\left(x + \frac{\pi}{2}\right)\right] + \frac{4}{5\pi}\sin\left[5\left(x + \frac{\pi}{2}\right)\right] + \ldots$$

$$f(x) = \frac{4}{\pi}\sin\left(x + \frac{\pi}{2}\right) + \frac{4}{3\pi}\sin\left(3x + \frac{3\pi}{2} - 2\pi\right) + \ldots$$

$$f(x) = \frac{4}{\pi}\sin\left(x + \frac{\pi}{2}\right) + \frac{4}{3\pi}\sin\left(3x - \frac{\pi}{2}\right) + \frac{4}{5\pi}\sin\left(5x + \frac{\pi}{2}\right) + \ldots$$

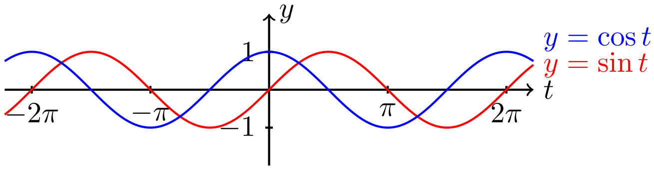
$$f(x) = \frac{4}{\pi}\cos(x) - \frac{4}{3\pi}\cos(3x) + \frac{4}{5\pi}\cos(5x) - \ldots$$

wobei folgende trigonometrischen Beziehungen gebraucht wurden:

$$\sin\left(\alpha + \frac{\pi}{2}\right) = \cos\alpha$$
$$\sin\left(\alpha - \frac{\pi}{2}\right) = -\cos\alpha$$

Offenbar gilt: $f(x) = g\left(x + \frac{\pi}{2}\right)$

Die Fourierreihe von $g(x)$ wurde in Aufgabe 9 (a) bestimmt.



Zur Erinnerung:

- $\sin\left(t + \frac{\pi}{2}\right) = \cos t$
- $\sin\left(t - \frac{\pi}{2}\right) = -\cos t$

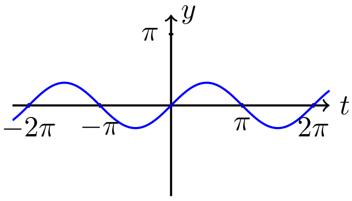
$$f(x) = \frac{4}{\pi}\sin\left(x + \frac{\pi}{2}\right) + \frac{4}{3\pi}\sin\left[3\left(x + \frac{\pi}{2}\right)\right] + \dots$$

$$f(x) = \frac{4}{\pi}\sin\left(x + \frac{\pi}{2}\right) + \frac{4}{3\pi}\sin\left(3x + \frac{3\pi}{2} - 2\pi\right)$$
$$+ \frac{4}{5\pi}\sin\left(5x + \frac{5\pi}{2} - 2\pi\right) + \frac{4}{7\pi}\sin\left(7x + \frac{7\pi}{2} - 4\pi\right) + \dots$$

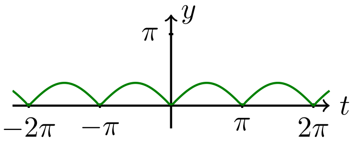
$$f(x) = \frac{4}{\pi}\sin\left(x + \frac{\pi}{2}\right) + \frac{4}{3\pi}\sin\left(3x - \frac{\pi}{2}\right)$$
$$+ \frac{4}{5\pi}\sin\left(5x + \frac{\pi}{2}\right) + \frac{4}{7\pi}\sin\left(7x - \frac{\pi}{2}\right) + \dots$$

$$f(x) = \frac{4}{\pi}\cos x - \frac{4}{3\pi}\cos 3x + \frac{4}{5\pi}\cos 5x - \frac{4}{7\pi}\cos 7x + \dots$$

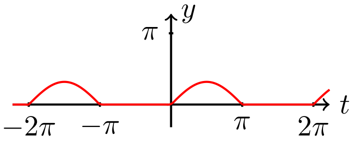
(d) Graph:



$y = g(x)$



$y = h(x)$



$y = \frac{1}{2}\big(g(x) + h(x)\big)$

$$f(x) = \frac{1}{2} \cdot \big(|\sin x| + \sin x\big)$$

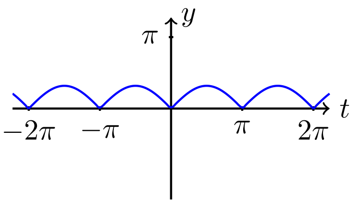
Die Fourierreihe von $g(x) = \big|\sin x\big|$ wurde in Aufgabe 9b bestimmt.

Die Fourierreihe von $h(x) = \sin x$ ist einfach $\sin x$

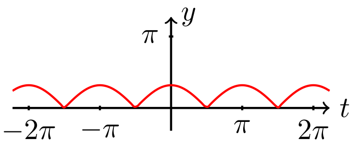
$$f(x) = \frac{1}{2}\left[\frac{2}{\pi} - \frac{4}{3\pi}\cos(2x) - \frac{4}{15\pi}\cos(4x) - \frac{4}{35\pi}\cos(6x) - \cdots + \sin x\right]$$

$$f(x) = \frac{1}{\pi} + \frac{1}{2}\sin x - \frac{2}{3\pi}\cos(2x) - \frac{2}{15\pi}\cos(4x) - \frac{2}{35\pi}\cos(6x) - \ldots$$

(e) Graph



$y = |\sin(x)|$



$y = |\cos x| = \left|\sin\left(x + \frac{\pi}{2}\right)\right|$

$$f(x) = \left|\sin\left(x - \frac{\pi}{2}\right)\right|$$

Die Fourierreihe von $g(x) = |\sin x|$ wurde in Aufgabe 3b bestimmt.

$$f(x) = \frac{2}{\pi} - \frac{4}{3\pi}\cos\left[2\left(x - \frac{\pi}{2}\right)\right] - \frac{4}{15\pi}\cos\left[4\left(x - \frac{\pi}{2}\right)\right] - \ldots$$

$$f(x) = \frac{2}{\pi} - \frac{4}{3\pi}\cos(2x - \pi) - \frac{4}{15\pi}\cos(4x - 2\pi) - \frac{4}{35\pi}\cos(6x - 3\pi) - \ldots$$

$$f(x) = \frac{2}{\pi} + \frac{4}{3\pi}\cos(2x) - \frac{4}{15\pi}\cos(4x) + \frac{4}{35\pi}\cos(6x) - \ldots$$

wobei folgende trigonometrischen Beziehungen gebraucht wurden:

$$\cos\big(\alpha - \pi\big) = -\cos\alpha$$
$$\cos\big(\alpha - 2\pi\big) = \cos\alpha$$