Aufgabe 5.1

$f(x) = \sin x \quad \Rightarrow \quad f'(x) = \cos x$

$$s = \int_0^{2\pi} \sqrt{1 + \cos^2 x}\, \mathrm{d}x \overset{\text{TR}}{\approx} 7.640\,\text{LE}$$

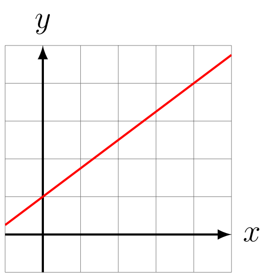
Aufgabe 5.2

$f(x) = \frac{1}{2}\mathrm{e}^x + \frac{1}{2}\mathrm{e}^{-x} \quad \Rightarrow \quad f'(x) = \frac{1}{2}\mathrm{e}^x - \frac{1}{2}\mathrm{e}^{-x} = \frac{1}{2}\left(\mathrm{e}^x - \mathrm{e}^{-x}\right)$

$$\begin{aligned}
s &= \int_0^1 \sqrt{1 + \frac{1}{4}\left(\mathrm{e}^x - \mathrm{e}^{-x}\right)^2}\, \mathrm{d}x \\
&= \frac{1}{2}\int_0^1 \sqrt{4 + \left(\mathrm{e}^{2x} - 2\mathrm{e}^0 + \mathrm{e}^{-2x}\right)}\, \mathrm{d}x \\
&= \frac{1}{2}\int_0^1 \sqrt{\mathrm{e}^{2x} + 2\mathrm{e}^0 + \mathrm{e}^{-2x}}\, \mathrm{d}x = \frac{1}{2}\int_0^1 \sqrt{\left(\mathrm{e}^x + \mathrm{e}^{-x}\right)^2}\, \mathrm{d}x \\
&= \frac{1}{2}\int_0^1 \left(\mathrm{e}^x + \mathrm{e}^{-x}\right) \mathrm{d}x = \frac{1}{2}\left[\mathrm{e}^x - \mathrm{e}^{-x}\right]_0^1 \\
&= \frac{1}{2}\left[\left(\mathrm{e} - \mathrm{e}^{-1}\right) - (1 - 1)\right] = \frac{1}{2}\left(\mathrm{e} - \mathrm{e}^{-1}\right) \approx 1.175
\end{aligned}$$

Aufgabe 5.3

Graph:

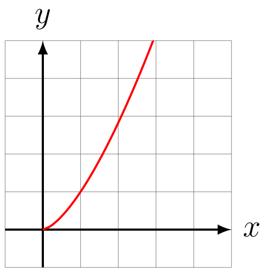


$f(x) = \frac{3}{4}x + 1 \quad \Rightarrow \quad f'(x) = \frac{3}{4}$

$$s = \int_0^4 \sqrt{1 + \left(\frac{3}{4}\right)^2}\, \mathrm{d}x = \int_0^4 \sqrt{\frac{25}{16}}\, \mathrm{d}x = \int_0^4 \frac{5}{4}\, \mathrm{d}x = \left[\frac{5}{4}x\right]_0^4 = 5$$

Aufgabe 5.4

Graph:



$f(x) = \sqrt{x^3} = x^{\frac{3}{2}} \quad \Rightarrow \quad f'(x) = \frac{3}{2}x^{\frac{1}{2}}$

Achtung: In der Aufgabe hätte als obere Grenze $x = 4$ stehen sollen

$$s = \int_0^4 \sqrt{1 + \left(\frac{3}{2}x^{\frac{1}{2}}\right)^2}\,\mathrm{d}x = \int_0^4 \sqrt{1 + \frac{9}{4}x}\,\mathrm{d}x = \dots$$

$$u = 1 + \frac{9}{4}x \quad \Rightarrow \quad \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{9}{4}$$

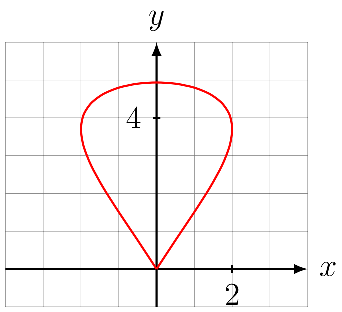
$$\mathrm{d}x = \frac{4}{9}\,\mathrm{d}u$$

Grenzen: $u(0) = 1$
$u(4) = 10 \quad [u(9) = 85/4]$

$$\dots = \frac{4}{9}\int_1^{10} \sqrt{u}\,\mathrm{d}u = \frac{4}{9}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_1^{10} = \frac{8}{27}\left(10\sqrt{10} - 1\right) \approx 9.073$$

Mit der „weniger schönen“ oberen Grenze $x = 9$ erhält man $s = \dfrac{1}{27}\left(85\sqrt{85} - 8\right) \approx 28.728$.

Aufgabe 5.5

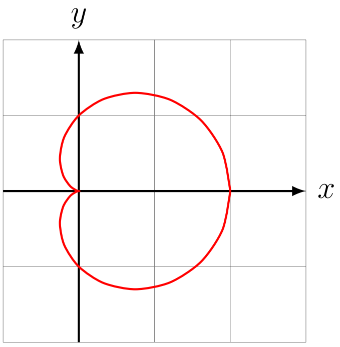


$x(t) = 2\sin t \quad \Rightarrow \quad \dot{x}(t) = 2\cos t$

$y(t) = \pi t - t^2/2 \quad \Rightarrow \quad \dot{y}(t) = \pi - t$

$$b = \int_0^{2\pi} \sqrt{\dot{x}(t)^2 + \dot{y}(t)^2}\,\mathrm{d}t = \int_0^{2\pi} \sqrt{4\cos^2(t) + (\pi - t)^2}\,\mathrm{d}t \approx 13.73$$

Aufgabe 5.6



$r(\varphi) = 1 + \cos\varphi$

$r'(\varphi) = -\sin\varphi$

$$b = \int_0^{2\pi} \sqrt{\left[r(\varphi)\right]^2 + \left[r'(\varphi)\right]^2}\,\mathrm{d}\varphi = \sqrt{(1+\cos(\varphi))^2 + (\sin\varphi)^2}\,\mathrm{d}\varphi = 8$$

Aufgabe 5.7

$f(x) = \ln(\sin x)$

$f'(x) = \frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x}$

$$s = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \sqrt{1 + \frac{\cos^2 x}{\sin^2 x}}\,\mathrm{d}x = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \sqrt{\frac{\sin^2 x + \cos^2 x}{\sin^2 x}}\,\mathrm{d}x$$

$$= \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \sqrt{\frac{1}{\sin^2 x}}\,\mathrm{d}x = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{1}{\sin x}\,\mathrm{d}x \overset{\text{FTB}}{=} 4\left[\ln\left|\tan\frac{x}{2}\right|\right]_{\frac{\pi}{3}}^{\frac{2\pi}{3}}$$

$$= \ln\tan\frac{\pi}{3} - \ln\tan\frac{\pi}{6} = \ln\sqrt{3} - \ln\frac{1}{\sqrt{3}} = \ln\sqrt{3} + \ln\sqrt{3}$$

$$= 2\ln\sqrt{3} = \ln 3 \approx 1.099$$