

Aufgabe 3.1

$$x^4 - 3x^2 - 4 = (x^2 - 4)(x^2 + 1) = 0$$

$$x^2 = 4 \quad \Rightarrow \quad x_1 = -2, x_2 = 2$$

$$x^2 = -1 \quad \Rightarrow \quad x_3 = -i, x_4 = i$$

Aufgabe 3.2

Ansatz: $z = x + iy$

$$2z - i(z + 2 + 6i) = \bar{z}$$

$$2(x + iy) - i(x + iy + 2 + 6i) = x - iy$$

$$2x + 2yi - xi + y - 2i + 6 = x - yi$$

Gleichung der Realteile: $2x + y + 6 = x$

$$x + y = -6 \quad [1]$$

Gleichung der Imaginärteile: $2y - x - 2 = -y$

$$-x + 3y = 2 \quad [2]$$

$$[1]+[2]: 4y = -4 \quad \Rightarrow \quad y = -1$$

$$[1]: x - 1 = -6 \quad \Rightarrow \quad x = -5$$

$$z = -5 - i$$

Aufgabe 3.3

$$z^3 = 1 = \text{cis } 0^\circ$$

$$z_0 = \text{cis } \frac{0^\circ + 0 \cdot 360^\circ}{3} = \text{cis } 0^\circ = 1$$

$$z_1 = \text{cis } \frac{0^\circ + 1 \cdot 360^\circ}{3} = \text{cis } 120^\circ = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$z_2 = \text{cis } \frac{0^\circ + 2 \cdot 360^\circ}{3} = \text{cis } 240^\circ = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

Aufgabe 3.4

$$z^4 = 16i = 16 \operatorname{cis} 90^\circ = 2^4 \operatorname{cis} 90^\circ$$

$$z_0 = 2 \operatorname{cis} \frac{90^\circ + 0 \cdot 360^\circ}{4} = 2 \operatorname{cis} 22.5^\circ$$

$$z_1 = 2 \operatorname{cis} \frac{90^\circ + 1 \cdot 360^\circ}{4} = 2 \operatorname{cis} 112.5^\circ$$

$$z_2 = 2 \operatorname{cis} \frac{90^\circ + 2 \cdot 360^\circ}{4} = 2 \operatorname{cis} 202.5^\circ$$

$$z_3 = 2 \operatorname{cis} \frac{90^\circ + 3 \cdot 360^\circ}{4} = 2 \operatorname{cis} 292.5^\circ$$

Aufgabe 3.5

$$z^2 = 1 + \sqrt{3}i = 2 \operatorname{cis} 60^\circ$$

$$z_0 = \sqrt{2} \operatorname{cis} \frac{60^\circ + 0 \cdot 360^\circ}{2} = \sqrt{2} \operatorname{cis} 30^\circ$$

$$= \sqrt{2}(\cos 30^\circ + i \sin 30^\circ) = \frac{\sqrt{6}}{2} + i \frac{\sqrt{2}}{2}$$

$$z_2 = \sqrt{2} \operatorname{cis} \frac{60^\circ + 1 \cdot 360^\circ}{2} = \sqrt{2} \operatorname{cis} 210^\circ$$

$$= \sqrt{2}(\cos 210^\circ + i \sin 210^\circ) = -\frac{\sqrt{6}}{2} - i \frac{\sqrt{2}}{2}$$

Aufgabe 3.6

$$z + 3 = 2i(z - 4i)$$

$$z + 3 = 8 + 2zi$$

$$z - 2zi = 5$$

$$z(1 - 2i) = 5$$

$$z = \frac{5}{1 - 2i}$$

$$= \frac{5(1 + 2i)}{(1 + 2i)(1 - 2i)} = \frac{5(1 + 2i)}{5} = 1 + 2i$$

Aufgabe 3.7

Setze $z = x + iy$

$$\operatorname{Re}(z) + 2iz + 3\bar{z} = 8$$

$$x + 2i(x + iy) + 3(x - iy) = 8$$

$$x + 2xi - 2y + 3x - 3iy = 8$$

$$4x - 2y + 2xi - 3yi = 8$$

Koeffizientenvergleich: $4x - 2y = 8$ [1] Realteile
 $2x - 3y = 0$ [2] Imaginärteile

$$[1] - 2 \cdot [2]: 4y = 8 \Rightarrow y = 2$$

$$[2]: 2x - 6 = 0 \Rightarrow x = 3$$

$$z = 3 + 2i$$

Aufgabe 3.8

x		-9	25	-25	
1	1	-8	17	-8	$(x = 1 \text{ ist keine Nullstelle})$
5	1	-4	5	0	$(x_1 = 5 \text{ ist erste Nullstelle})$

Faktorzerlegung:

$$x^3 - 9x^2 + 25x - 25 = (x - 5)(x^2 - 4x + 5)$$

$$D = b^2 - 4ac = 16 - 4 \cdot 1 \cdot 5 = -4$$

$$x_2 = \frac{-b + \sqrt{D}}{2a} = \frac{4 + 2i}{2} = 2 + i$$

$$x_3 = \frac{-b - \sqrt{D}}{2a} = \frac{4 - 2i}{2} = 2 - i$$

Aufgabe 3.9

$$z^2 - (6 - 2i)z + (11 - 10i) = 0$$

Koeffizienten: $a = 1$, $b = -(6 - 2i)$ $c = 11 - 10i$

$$d^2 = b^2 - 4ac = (6 - 2i)^2 - 4 \cdot 1 \cdot (11 - 10i)$$

$$= 32 - 24i - 44 + 40i = -12 + 16i$$

$$\text{Ansatz: } d^2 = (x + iy)^2 = x^2 - y^2 + 2xyi = -12 + 16i$$

$$\text{Koeffizientenvergleich: } x^2 - y^2 = -12 \Rightarrow \begin{aligned} d_1 &= 2 + 4i \\ 2xy &= 16 \quad d_2 = -2 - 4i \end{aligned}$$

$$z_1 = \frac{-b + d_1}{2a} = \frac{6 - 2i + 2 + 4i}{2} = 4 + i$$

$$z_2 = \frac{-b + d_2}{2a} = \frac{6 - 2i - 2 - 4i}{2} = 2 - 3i$$

Aufgabe 3.10

$$\text{Normierte Form: } x^3 - 3x^2 + 4 = 0$$

$$p = s - r^2/3 = -3$$

$$q = 2r^3/27 - rs/3 + t = 2$$

$$\text{Reduzierte Form: } y^3 - 3y + 2 = 0$$

$$D = (p/3)^3 + (q/2)^2 = 0$$

$$y_1 = -\sqrt[3]{4q} = -2$$

$$y_2 = \sqrt[3]{q/2} = 1$$

$$y_3 = \sqrt[3]{q/2} = 1$$

$$x_1 = y_1 - r/3 = -1$$

$$x_2 = y_2 - r/3 = 2$$

$$x_3 = y_3 - r/3 = 2$$

Aufgabe 3.11

$$\text{Normierte Form: } x^3 + 6x^2 + 4x - 8 = 0$$

$$r = 6, s = 4, t = -8$$

$$p = s - r^2/3 = -8$$

$$q = 2r^3/27 - rs/3 + t = 0$$

$$\text{Reduzierte Form: } y^3 - 8y = y(y^2 - 8) = 0$$

$$y_1 = 0$$

$$y_2 = 2\sqrt{2}$$

$$y_3 = -2\sqrt{2}$$

$$x_1 = y_1 - r/3 = -2$$

$$x_2 = y_2 - r/3 = -2 + 2\sqrt{2}$$

$$x_3 = y_3 - r/3 = -2 - \sqrt{2}$$

Aufgabe 3.12

$$\text{Allgemeine Form: } 2x^3 - 3x^2 + 6x - 9 = 0$$

$$r = b/a = -3/2, s = c/a = 3, t = d/a = -9/2$$

$$\text{Normierte Form: } x^3 - 3/2x^2 + 3x - 9/2 = 0$$

$$p = s - r^2/3 = 9/4$$

$$q = 2r^3/27 - rs/3 + t = -13/4$$

Reduzierte Form: $y^3 + 9/4y - 13/4 = 0$

$$D = (p/3)^3 + (q/2)^2 = 49/16$$

$$u = \frac{3}{2}$$

$$v = -\frac{1}{2}$$

$$y_1 = u + v = 1$$

$$y_2 = -\frac{u+v}{2} + \frac{u-v}{2}\sqrt{3}i = -\frac{1}{2} + \sqrt{3}i$$

$$y_3 = -\frac{u+v}{2} - \frac{u-v}{2}\sqrt{3}i = -\frac{1}{2} - \sqrt{3}i$$

$$x_1 = y_1 - \frac{r}{3} = \frac{3}{2}$$

$$x_2 = y_2 - \frac{r}{3} = \sqrt{3}i$$

$$x_3 = y_3 - \frac{r}{3} = -\sqrt{3}i$$

Aufgabe 3.13

- 6 reelle Lösungen
- 4 reelle Lösungen und 1 Paar konjugiert komplexer Lösungen
- 2 reelle Lösungen und 2 Paare konjugiert komplexer Lösungen
- 3 Paare konjugiert komplexer Lösungen