

Aufgabe 2.1

(a) $z = 5i = 5 \operatorname{cis}(90^\circ)$

(b) $z = -4 + 4i$

$$|z| = \sqrt{(-4)^2 + 4^2} = \sqrt{2 \cdot 16} = 4\sqrt{2}$$

$$\begin{aligned} \arg(z) &= \arctan\left(-\frac{4}{4}\right) + 180^\circ = -\arctan(1) + 180^\circ \\ &= -45^\circ + 180^\circ = 135^\circ \end{aligned}$$

$$z = 4\sqrt{2} \operatorname{cis}(135^\circ)$$

(c) $z = -2\sqrt{3} - 6i$

$$|z| = \sqrt{(-2\sqrt{3})^2 + (-6)^2} = \sqrt{12 + 36} = \sqrt{48} = 4\sqrt{3}$$

$$\begin{aligned} \arg(z) &= \arctan\left(\frac{6}{2\sqrt{3}}\right) + 180^\circ = \arctan\left(\frac{6\sqrt{3}}{6}\right) + 180^\circ \\ &= \arctan(\sqrt{3}) + 180^\circ = 60^\circ + 180^\circ = 240^\circ \end{aligned}$$

$$z = 4\sqrt{3} \operatorname{cis}(240^\circ)$$

(d) $z = 3 - \sqrt{3}i$

$$|z| = \sqrt{3^2 + (-\sqrt{3})^2} = \sqrt{9 + 3} = \sqrt{12} = 2\sqrt{3}$$

$$\arg(z) = \arctan\left(-\frac{\sqrt{3}}{3}\right) = -\arctan\left(\frac{\sqrt{3}}{3}\right) = -30^\circ$$

$$z = 2\sqrt{3} \operatorname{cis}(330^\circ)$$

Aufgabe 2.2

(a) $5.7 \operatorname{cis}(\pi) = 5.7 \cdot (-1) = -5.7$

(b) $3 \operatorname{cis}(225^\circ) = 3 \cos(225^\circ) + 3 \sin(225^\circ)$

$$= 3 \cos(130^\circ + 45^\circ) + 3 \sin(180^\circ + 45^\circ)$$

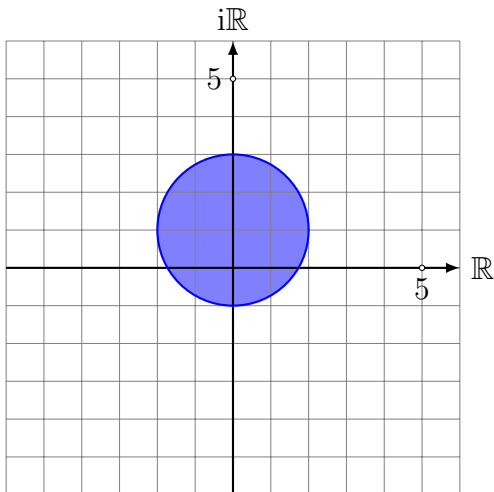
$$= -3 \cos(45^\circ) - 3i \sin(45^\circ)$$

$$= -3 \cdot \frac{\sqrt{3}}{2} - 3 \cdot \frac{\sqrt{2}}{2}i = -\frac{3\sqrt{3}}{2} - \frac{3\sqrt{2}}{2}i$$

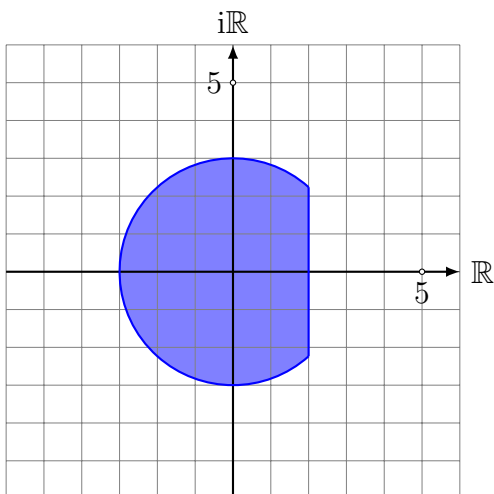
$$\begin{aligned}
(c) \quad 4 \operatorname{cis}(5\pi/6) &= 4 \cos(150^\circ) \\
&= 4 \cos(180^\circ - 30^\circ) + 4i \sin(180^\circ - 30^\circ) \\
&= -4 \cos(30^\circ) + 4i \sin(30^\circ) = -4 \cdot \frac{\sqrt{3}}{2} + 4i \cdot \frac{1}{2} \\
&= -2\sqrt{3} + 2i
\end{aligned}$$

$$\begin{aligned}
(d) \quad 6\sqrt{3} \operatorname{cis}(300^\circ) &= 6\sqrt{3} \cos(360^\circ - 60^\circ) + 6\sqrt{3}i \sin(360^\circ - 60^\circ) \\
&= -6\sqrt{3} \cos(60^\circ) + 6\sqrt{3}i \sin(60^\circ) \\
&= -6\sqrt{3} \cdot \frac{1}{2} + 6\sqrt{3}i \cdot \frac{\sqrt{3}}{2} = 3\sqrt{3} - 9i
\end{aligned}$$

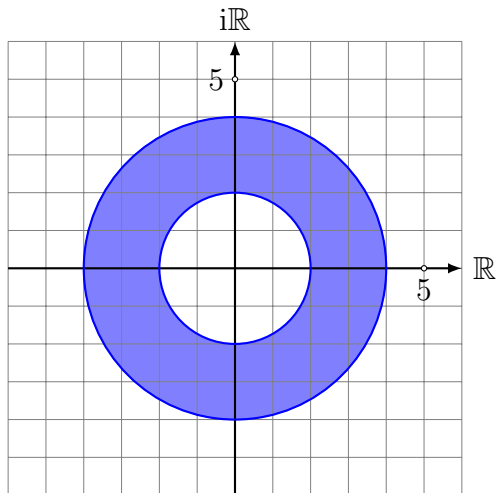
Aufgabe 2.3



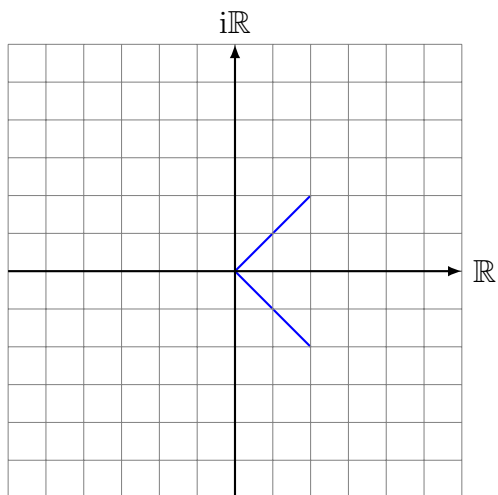
Aufgabe 2.4



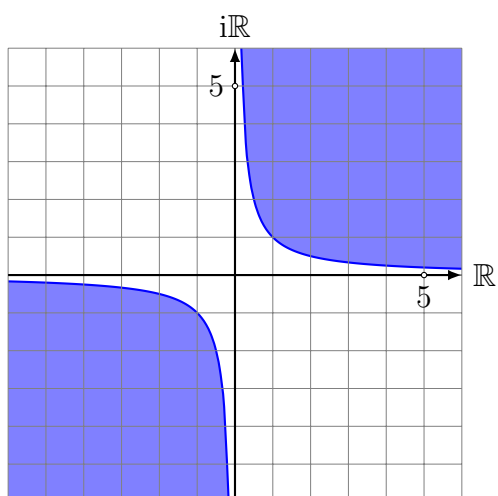
Aufgabe 2.5



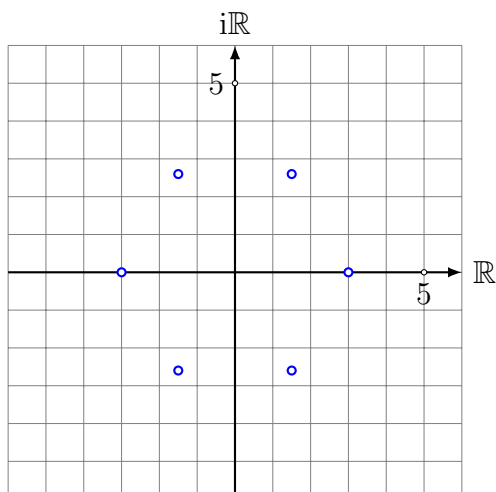
Aufgabe 2.6



Aufgabe 2.7



Aufgabe 2.8



Aufgabe 2.9

- (a) $4 \operatorname{cis} 20^\circ \cdot 5 \operatorname{cis} 30^\circ = 20 \operatorname{cis} 50^\circ$
- (b) $9 \operatorname{cis} 40^\circ : 2 \operatorname{cis} 70^\circ = 4.5 \operatorname{cis}(-30^\circ) = 4.5 \operatorname{cis} 330^\circ$
- (c) $(\sqrt{2} \operatorname{cis} 40^\circ)^{10} = \sqrt{2}^{10} \operatorname{cis} 400^\circ = 2^5 \operatorname{cis} 40^\circ = 32 \operatorname{cis} 40^\circ$
- (d) $(\operatorname{cis} 12^\circ)^{15} \cdot (\operatorname{cis} 15^\circ)^{12} = \operatorname{cis} 180^\circ \cdot \operatorname{cis} 180^\circ = \operatorname{cis} 360^\circ = 1$

Aufgabe 2.10

- (a) $\operatorname{cis} \frac{5\pi}{6} \cdot \operatorname{cis} \frac{4\pi}{3} = \operatorname{cis} \frac{13\pi}{6} = \operatorname{cis} \left(\frac{13\pi}{6} - 2\pi \right) = \operatorname{cis} \frac{\pi}{6}$
- (b) $\operatorname{cis}^{-5} \left(-\frac{2\pi}{3} \right) = \operatorname{cis} \frac{10\pi}{3} = \operatorname{cis} \left(\frac{10\pi}{3} - 2\pi \right) = \operatorname{cis} \frac{4\pi}{3}$
- (c) $\prod_{k=1}^{10} \operatorname{cis} \frac{k\pi}{4} = \operatorname{cis} \frac{(1+2+\dots+10)\pi}{4} = \operatorname{cis} \frac{55\pi}{4} = \operatorname{cis} \frac{7\pi}{4}$

Aufgabe 2.11

- (a) $\operatorname{cis} \varphi \cdot \operatorname{cis}(-\varphi) = \operatorname{cis} [\varphi + (-\varphi)] = \operatorname{cis} 0 = 1$
- (b) $\operatorname{cis} \varphi - \operatorname{cis}(-\varphi) = [\cos \varphi + i \sin \varphi] - [\cos(-\varphi) + i \sin(-\varphi)]$
 $= [\cos \varphi + i \sin \varphi] - [\cos \varphi - i \sin \varphi]$
 $= 2i \sin \varphi$

Aufgabe 2.12

- (a) $(\cos 25^\circ - i \sin 25^\circ)(\cos 35^\circ - i \sin 35^\circ)$
 $= (\cos(-25^\circ) + i \sin(-25^\circ))(\cos(-35^\circ) + i \sin(-35^\circ))$
 $= \operatorname{cis}(-25^\circ) \operatorname{cis}(-35^\circ) = \operatorname{cis}(-60^\circ)$
 $= \operatorname{cis}(-60^\circ + 360^\circ) = \operatorname{cis} 300^\circ$
- (b) $\frac{\cos 40^\circ + i \sin 40^\circ}{\cos 30^\circ - i \sin 30^\circ} = \frac{\operatorname{cis} 40^\circ}{\cos(-30^\circ) + i \sin(-30^\circ)}$
 $= \frac{\operatorname{cis} 40^\circ}{\operatorname{cis}(-30^\circ)} = \operatorname{cis} 70^\circ$

Aufgabe 2.13

$$\operatorname{cis} 2\varphi = (\operatorname{cis} \varphi)^2$$
$$\operatorname{cis} 2\varphi = (\cos \varphi + i \sin \varphi)(\cos \varphi + i \sin \varphi)$$
$$\cos 2\varphi + i \sin 2\varphi = \cos^2 \varphi - \sin^2 \varphi + i \cdot 2 \sin \varphi \cos \varphi$$

Durch den Vergleich von Real- und Imaginärteil erhält man:

$$\cos 2\varphi = \cos^2 \varphi - \sin^2 \varphi$$

$$\sin 2\varphi = 2 \sin \varphi \cos \varphi$$

Aufgabe 2.14

- (a) $3e^{i\frac{\pi}{6}} \cdot 2e^{i\frac{\pi}{3}} = 6e^{i(\frac{\pi}{6} + \frac{\pi}{3})} = 6e^{i\frac{\pi}{2}} = 6i$
- (b) $3e^{i\frac{17\pi}{5}} : 2e^{i\frac{12\pi}{5}} = 1.5e^{i(\frac{17\pi}{5} - \frac{12\pi}{5})} = 1.5e^{i\pi} = -1.5$
- (c) $(e^{-i\frac{3\pi}{2}})^4 = e^{-i\frac{12\pi}{2}} = e^{i2\pi} = 1$

Aufgabe 2.15

(a) $i^i = (e^{i\pi/2})^i = e^{i^2 \pi/2} = e^{-\pi/2}$

(b) $\sqrt{i} = \sqrt{e^{i\pi/2}} = (e^{i\pi/2})^{1/2} = e^{i\pi/4}$
 $= \cos \frac{\pi}{4} + i \frac{\pi}{4} = \frac{\sqrt{2}}{2} + i \frac{\pi}{4}$

(c) $\ln(1+i) = \ln(\sqrt{2}e^{i\pi/4}) = \ln(\sqrt{2}) + \ln(e^{i\pi/4}) = \ln(\sqrt{2}) + i\frac{\pi}{4}$

$$\begin{aligned}
\text{(d) } \sin(1+i) &= \frac{1}{2i}(e^{i(1+i)} - e^{-i(1+i)}) = \frac{1}{2i}(e^{i-1} - e^{-i+1}) \\
&= \frac{1}{2i}(e^i e^{-1} - e^{-i} e) = \frac{1}{2i}(e^{-1} e^i - e e^{-i}) \\
&= \frac{e^{-1}}{2i}(\cos 1 + i \sin 1) - \frac{e}{2i}(\cos 1 - i \sin 1) \\
&= \frac{e^{-1} - e}{2i} \cos 1 + i \frac{e^{-1} + e}{2i} \sin 1 \\
&= \frac{e^{-1} + e}{2} \sin 1 - i \frac{e^{-1} - e}{2} \cos 1 \\
&= \frac{e + e^{-1}}{2} \sin 1 + i \frac{e - e^{-1}}{2} \cos 1 \\
&[= \cosh(1) \sin(1) + i \sinh(1) \cos(1)]
\end{aligned}$$