

Aufgabe 10.1

$$f^{(0)}(x) = \cos(x) \quad \Rightarrow \quad f^{(0)}(\pi) = \cos(\pi) = -1$$

$$f^{(1)}(x) = -\sin(x) \quad \Rightarrow \quad f^{(1)}(\pi) = -\sin(\pi) = 0$$

$$f^{(2)}(x) = -\cos(x) \quad \Rightarrow \quad f^{(2)}(\pi) = -\cos(\pi) = 1$$

$$f^{(3)}(x) = \sin(x) \quad \Rightarrow \quad f^{(3)}(\pi) = \sin(\pi) = 0$$

$$f^{(4)}(x) = \cos(x) \quad \Rightarrow \quad f^{(4)}(\pi) = \cos(\pi) = -1$$

$$\cos(x) \approx -1 + \frac{1}{2!}(x - \pi)^2 - \frac{1}{4!}(x - \pi)^4$$

Aufgabe 10.2

$$(a) \quad f^0(x) = \frac{1}{x+1} \quad \Rightarrow \quad f^0(0) = 1$$

$$f^1(x) = -\frac{1}{(1+x)^2} \quad \Rightarrow \quad f^1(0) = -1$$

$$\frac{1}{x+1} \approx 1 - x$$

(b) Formelsammlung (S. 64):

- exakter Wert: $a = \frac{1}{0.01 + 1} = \frac{1}{101/100} = \frac{100}{101}$

- Näherung: $a_0 = 1 - 0.01 = 0.99 = \frac{99}{100}$

- absoluter Fehler: $\varepsilon_a = a_0 - a = \frac{99}{100} - \frac{100}{101} = \frac{-1}{10100}$

- relativer Fehler: $\frac{\varepsilon_a}{a} = \frac{101}{100} \cdot \frac{-1}{101000} = -10^{-4}$

Aufgabe 10.3

$$(a) \quad \frac{\sin x - x \cos x}{x^2}$$

$$= \frac{1}{x^2} \left[\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \right) - x \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots \right) \right]$$

$$= \frac{1}{x^2} \left[-\frac{x^3}{3!} + \frac{x^3}{2!} + \frac{x^5}{5!} - \frac{x^5}{4!} - \dots \right]$$

$$= \left[-\frac{x}{3!} + \frac{x}{2!} + \frac{x^3}{5!} - \frac{x^3}{4!} - \dots \right] \rightarrow 0 \quad \text{wenn } x \rightarrow 0$$

Aufgabe 10.3

$$\begin{aligned} \text{(b)} \quad \frac{\ln(1+x)}{x} &= \frac{1}{x} \cdot \left[x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \right] \\ &= 1 - \frac{x}{2} + \frac{x^2}{3} - \dots \rightarrow 1 \quad \text{für } x \rightarrow 0 \end{aligned}$$

Aufgabe 10.4

$$\arcsin(x) = e^x$$

$$x \approx 1 - x$$

$$2x \approx 1$$

$$x \approx 0.5$$

Aufgabe 10.5

$$p(1+x) = p^{(0)}(1) + p^{(1)}(1)x + \frac{p^{(2)}(1)}{2!}x^2 + \frac{p^{(3)}(1)}{3!}x^3 + \frac{p^{(4)}(1)}{4!}x^4$$

$$p^{(0)}(x) = x^4 + 2x^3 - x + 4 \quad \Rightarrow \quad p^{(0)}(1) = 6$$

$$p^{(1)}(x) = 4x^3 + 6x^2 - 1 \quad \Rightarrow \quad p^{(1)}(1) = 9$$

$$p^{(2)}(x) = 12x^2 + 12x \quad \Rightarrow \quad p^{(2)}(1) = 24$$

$$p^{(3)}(x) = 24x + 12 \quad \Rightarrow \quad p^{(3)}(1) = 36$$

$$p^{(4)}(x) = 24 \quad \Rightarrow \quad p^{(4)}(1) = 24$$

$$\begin{aligned} p(x+1) &= 6 + 9x + \frac{24}{2!}x^2 + \frac{36}{3!}x^3 + \frac{24}{4!}x^4 \\ &= x^4 + 6x^3 + 12x^2 + 9x + 6 \end{aligned}$$